

# **International GCSE Mathematics**

**A guide to teaching  
transferable skills**

# Contents

**Page 3. Introduction**

**Page 4. Strategies for teaching transferable skills**

**Page 8. Appendix 1:** Mapping grid for transferable skills for Maths A

**Page 15. Appendix 2:** Mapping grid for transferable skills for Maths B

**Page 20. Appendix 3:** Mapping grid for transferable skills for Further Pure Maths

**Page 27. Appendix 4:** 'Written relay' on Pythagoras' theorem and trigonometric facts

**Page 28. Appendix 5:** 'Written relay' on Quadrilaterals

**Page 29. Appendix 6:** Problem-solving with Pythagoras' theorem and trigonometry lesson plan

**Page 32. Appendix 7:** 'Paired coaching' worksheets

**Page 34. Appendix 8:** 'Question roundabout' using Pythagoras' theorem and trigonometry past exam questions

**Page 35. Appendix 9:** Number grids lesson plan

**Page 38. Appendix 10:** Number grid worksheet

*If printing this document we recommend printing in colour.*

# Introduction

Transferable skills will help students cope with the different demands of degree study and provide a solid skills base that enables them to adapt and thrive in different environments across educational stages; and ultimately into employment.

A good international education should enable students to start developing transferable skills as early as possible. Developing these transferable skills where they naturally occur as part of the International GCSE curriculum can help build learner confidence and embed the importance of this well-rounded development. This builds the foundations to ensure students are ready for A-level and higher education.

Our approach to enhancing transferable skills in our International GCSEs ensures that it is not only the academic and cognitive skills that are developed, but those broader elements that universities highlight as being essential for success. Skills such as self-directed study, independent research, self-awareness of own strengths and weaknesses and time-management are skills that students cannot learn from a textbook but have to be developed through the teaching and learning experience that can be provided through an international curriculum.

This guide is designed to provide you with ideas of different teaching approaches that will support the development of transferable skills.

The transferable skills referred to throughout this guide have been taken from the International GCSE specifications A and B and Further Pure Mathematics. This guide gives suggestions as to various strategies that could be used to teach the skills, including a couple of sample lesson plans. The tables (**Appendix 1**), give an explicit definition of how each skill can be interpreted for Mathematics. This will enable teachers and learners to understand examples of how they can develop each skill through this International GCSE. It also provides examples of where transferable skills could be covered in the content of the course and examples of where they are assessed in examinations.

# Strategies for teaching transferable skills

Pair or group work is a very effective way to teach students many transferable skills. In order to develop good team work skills it is very useful to use structured pair or group activities. The suggestions below may give you some ideas of the structures you could use. The advantages of using any of the suggestions here are that they all ensure that all students have a role throughout the activity.

## Verbal relay

This is useful for students who need to quickly recap on short facts. Students work in pairs to make a verbal list with no repeats, Student A goes first then B, then A again, etc. The teacher should randomly choose who is student A. This could be done by saying student A is the student with the biggest handspan/the tallest/who has the closest birthday to today/etc. An example of a 'verbal relay' is naming polygons and their number of sides.

## Written relay

This is useful for students who need to recap on facts. Students work in pairs to make a written list on a blank piece of paper with no repeats, Student A goes first then B, then A again, etc. The teacher should randomly choose who is student A. This could be done by saying student A is the eldest student/the student who woke up first today/etc. An example of a 'written relay' is types of numbers with examples or trigonometric facts. See Appendix 2 for two examples of written relays. The first example is on Pythagoras' theorem and trigonometric facts from two higher ability students aged 16 at the start of a revision lesson. The second example is on quadrilaterals from two middle ability students aged 10. See the first activity in the main part of the lesson in the lesson plan in Appendix 3 for an example of how to use a 'written relay'.

## Structured pair

Initially students are given a set amount of time to think the task through (the amount of time depends on the task and depending on the task, students may be allowed to make notes or draw diagrams to help).

Students then pair up to agree on a joint solution to the task, depending on the task, students could be allowed to make notes or draw diagrams. Each student then puts their solution in writing (this is done individually since they have already agreed on a solution in pairs).

### Structured group

Students work in a group of 4 to solve the task and they number themselves 1 to 4. When all groups have their solutions and explanations ready the teacher chooses one number. The student with that number is then the spokesperson for the group, they may be asked to stand, they then feed any results back to the teacher and the class (therefore, it is important that every student in each group understands the solutions as they don't know which student will be the spokesperson until the task is complete). See the starter in the lesson plan in Appendix 3 for an example of how to use a 'structured group'.

### Paired coaching

Students work in pairs to answer the questions on the worksheet. Student A answers their question 1, whilst student B watches them, checking their working as they go. When student A has completed question 1 student B says whether they think the solution is right or not. If they believe the answer is incorrect then they coach student A through where they have gone wrong so that they can correct their solution if necessary. When the students both agree that the solution is correct student B answers their question 1, whilst student A watches them, checking their working as they go. When student B has completed question 1 student A says whether they think the solution is right or not. If they believe the answer is incorrect then they coach student B through where they have gone wrong so that they can correct their solution if necessary. Then repeat this process for the rest of the worksheet. See the third activity in the main part of the lesson in the lesson plan in Appendix 3 for an example of how to use 'paired coaching'. See Appendix 4 for examples of 'paired coaching' worksheets. As can be seen in the worksheets given in Appendix 4, 'paired coaching' can be used to practice a method at any level of Maths and for any age group. The examples also demonstrate that 'paired coaching' can be used for repetitive exercises or mixed exercises (the first 'paired coaching' worksheet is a repetitive exercise on written methods of multiplication and the second 'paired coaching'

worksheet uses a variety of past exam questions on Pythagoras' theorem and trigonometry).

### Question relay

This is a variation on 'paired coaching'. This can be used as an 'exam relay' as well as with any exercise. Cut the questions out of the exam paper/exercise so that each question is on a separate piece of paper and put them into piles on your desk or a spare desk in the classroom. Students work in pairs to answer the questions using a relay style. Student A collects and answers any question, whilst student B watches them, checking their working as they go. When student A has completed question 1 student B says whether they think the solution is right or not. If they believe the answer is incorrect then they coach student A through where they have gone wrong so that they can correct their solution if necessary. When the students both agree that the solution is correct they can either go to get it marked by the teacher or 'store' it for marking at a later stage (this depends on how many students you have in your class, having 25+ students can make the instant marking an impossible task, you can end up with quite a long queue of students waiting for you which wastes their time). Now the process is repeated for student B, etc.

### Question roundabout

This can be used as an 'exam roundabout' as well as with any questions. Cut four questions out of the exam paper/exercise and glue them onto a piece of paper so that they all face outwards. See Appendix 5 for the example using four Pythagoras' theorem and trigonometry exam questions. The students are given a set amount of time to answer the question facing them. When the time is up students rotate the paper 90° clockwise. They now check the question facing them and make any amendments if necessary. This process is repeated until students have their starting question back. This could be done in pairs with just two questions on the sheet.

### Team coaching

This can be done using an exercise or exam paper. Cut the exercise or exam paper into individual questions so that each question is on a

separate piece of paper. Students work in groups of four. Each group is given a question to solve. One student is then selected to be the 'teacher' of the group. This student can be selected randomly by the teacher (the youngest student/the student with the most letters in their name/the student with the longest hair/etc) or nominated by the students. The student who is 'the teacher' stays with the solution to their question, ready to teach students from other groups. The remaining three students in each group visit the other 'teachers' to be coached through the solutions to the questions tackled by other groups. Students need to ensure that between them they visit every 'teacher'. Back in their starting groups, students then feedback how to tackle each question to the rest of their group. This is important as no-one in the group has seen every question. Students are now fully equipped to tackle the exam paper/exercise on their own. This may need to be done in the following lesson due to time restraints.

### Using open-ended tasks for teaching transferable skills

Open-ended tasks can be used to teach students transferable skills. An open-ended task is a task which students can extend according to their own ability and mathematical knowledge. Examples of open-ended tasks are mathematical investigations/problems, such as there are 10 people in a room, everyone shakes hands with everyone else, how many handshakes are there. See Appendix 6 for an example of a lesson plan using the handshake problem as a starter and a the number grid investigation as the main lesson. See Appendix 7 for the number grid investigation worksheet.

## Appendix 1: Mapping grid for transferable skills for Maths A

| Skill                              | Skill interpretation in this subject                                                                                                                                                     | Example of where the skill is covered in content                                                             | Example of where the skill is explicitly assessed in examination | Opportunity for the skill to be assessed formatively |
|------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------|------------------------------------------------------|
| <b>Cognitive skills</b>            |                                                                                                                                                                                          |                                                                                                              |                                                                  |                                                      |
| Cognitive Processes and Strategies |                                                                                                                                                                                          |                                                                                                              |                                                                  |                                                      |
| Critical thinking                  | Using <b>many</b> different pieces of mathematical information (sometimes seemingly unrelated) and synthesising this information to arrive at a solution to a mathematics-based problem. | e.g. 4.8F (3D trig and Pythagoras)<br><br>2.7D (Quadratic and linear equations)                              | e.g. 3H Qu 19 (4.8, 4.10)                                        | Yes                                                  |
| Problem solving                    | Translating problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes and solve them.                                                   | Most topics have some application here.<br>Explicitly 1.10a (Foundation)                                     | e.g. 1F Qu 15 (1.10)<br>2F qu 12 (4.9, 1.10)                     | Yes                                                  |
| Analysis                           | Examining and understanding different elements of a mathematical context or different mathematical processes.                                                                            | e.g. 3.3 (study of shape of graphs, turning points, roots etc. and relation to completing the square (2.2D)) | e.g. 4H Qu 22 (2.2d)<br><br>4H Qu 19 (3.3)                       | Yes                                                  |
| Reasoning                          | Making abstract deductions and draw conclusions from mathematical information.                                                                                                           | e.g. 4.7 (Geometrical reasoning especially using Circle theorems (4.6))<br>4.2C,D,E (sides of polygons)      | e.g. 3H Qu 16 (4.6, 4.7)                                         | Yes                                                  |

|                    |                                                                                                                                                 |                                                                                                                                                                                                                         |                                                         |                                                                    |
|--------------------|-------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------|--------------------------------------------------------------------|
| Interpretation     | Analysing mathematical information and understanding the meaning of that information, for example interpreting straight line conversion graphs. | Most topics cover this.<br>e.g. Conversion graphs (3.3G)<br>3.4E (Kinematics)<br>6.2 (Statistical measures)                                                                                                             | e.g. 4H Qu 25 (3.3G)<br><br>3H Qu 12 (6.2)              | Yes                                                                |
| Decision Making    | Selecting a mathematical process from a series of mathematical processes to solve a problem.                                                    | e.g. Selection of appropriate method in Trig and Pythagoras problems (4.8)<br>Use tools of algebra and statistics (6.2)                                                                                                 | e.g. 4H Qu 21 (4.8)<br><br>e.g. 2F Qu 22 (6.2)          | e.g. Use of discussion in whole class contexts or in small groups. |
| Adaptive learning  | Adapting a mathematical strategy to solve a context based mathematical problem.                                                                 | e.g. 1.6E, F percentage problems<br>1.7E ratio/proportion<br>2.3E deriving formulae                                                                                                                                     | e.g. 1F Qu 19 (1.6)<br>1F Qu 17 (1.7)<br>1F Qu 23 (1.6) | Yes                                                                |
| Executive function | Planning how to solve a problem, carrying out the plan and reviewing the outcome.                                                               | Principle of estimating an answer is in 1.8D which enables candidates to "review the outcome"<br>Questions in calculus (3.4C) to find turning points require candidates to select the appropriate stages (i.e. "plan"?) | e.g. 3H Qu 21 (3.4C)                                    | Yes                                                                |

| Creativity |                                                                                                                                                                                                                                                                    |                                                                                                                                                                                  |                                                                                                                                                                                                                                                                                                                                                                |                                                      |
|------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------|
| Creativity | <p>Using own learning to apply mathematical processes and link these together to prove and validate mathematical concepts (Although 'proof' may not really exist in Maths A).</p> <p>Uses a different, unexpected mathematical process to arrive at an answer.</p> | <p>We use "Show that" style of questions where candidates have to give something approaching a proof.</p> <p>Also 4.7A requires simple ideas of proof in geometric problems.</p> | <p>e.g. 3H Qus 13, 15, 18, 22</p> <p>2F Qu 25</p> <p>4H Qu 10</p>                                                                                                                                                                                                                                                                                              | <p>Yes</p> <p>May be evidenced in homework tasks</p> |
| Innovation | <p>Using a novel strategy to solve a previously unseen mathematical problem.</p>                                                                                                                                                                                   | <p>There is scope here in the area of turning points on curves (sections 3.3 and 3.4)</p>                                                                                        | <p>Hard to explicitly assess but candidates may produce solutions not on mark scheme. e.g. to find the x-coordinate of the minimum on <math>y = 3x^2 - 9x + 5</math> the candidate uses ideas of symmetry and the mid-point of the roots. They may then use a knowledge that the sum of the roots is <math>-\frac{b}{a}</math> to write down the answer as</p> | <p>Yes</p> <p>See example.</p>                       |

|  |  |  |                                                                               |  |
|--|--|--|-------------------------------------------------------------------------------|--|
|  |  |  | $\frac{1}{2} \times \frac{9}{3} = \frac{3}{2}$ rather<br>than using calculus. |  |
|--|--|--|-------------------------------------------------------------------------------|--|

| Skill                                | Skill interpretation in this subject                                                                                                                        | Example of where the skill is covered in content  | Where the skill is explicitly assessed in examination                       | Opportunity for the skill to be assessed formatively                                                                                                        |
|--------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------|-----------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Intrapersonal skills</b>          |                                                                                                                                                             |                                                   |                                                                             |                                                                                                                                                             |
| <b>Intellectual openness</b>         |                                                                                                                                                             |                                                   |                                                                             |                                                                                                                                                             |
| Adaptability                         | Ability to select and apply knowledge and understanding of mathematical processes (that which is not prompted or provided) to unseen mathematical problems. | Many questions would assess this                  | Yes<br>Any question where we do not specify the method to use e.g. 4H Qu 21 | Yes                                                                                                                                                         |
| Personal and social responsibility   | Using mathematical knowledge and skills to solve a problem for which one is accountable.                                                                    | 1.10 is all about applying number in everyday use |                                                                             | Yes<br>e.g. students monitoring their allowance                                                                                                             |
| Continuous learning                  | Planning and reflecting on own learning- setting goals and meeting them regularly                                                                           | N/A                                               | N/A                                                                         | Yes<br>Students identify areas where they need extra help or practice.                                                                                      |
| Intellectual interest and curiosity  | Identifying a problem under own initiative, planning a solution and carrying this out.                                                                      | e.g. the topic of sequences lends itself to this  |                                                                             | Yes<br>Student goes on to try and find a formula for the $n$ th term ( $= 2n^2$ ) Not on specification but a simple question student could ask and explore. |
| <b>Work ethic/ conscientiousness</b> |                                                                                                                                                             |                                                   |                                                                             |                                                                                                                                                             |
| Initiative                           | Using mathematical knowledge, independently (without guided                                                                                                 | N/A                                               | N/A                                                                         | Yes                                                                                                                                                         |

|                                                          |                                                                                                                            |                                                                            |                                                                                                                             |                                                                                                 |
|----------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------|
|                                                          | learning), to further own understanding.                                                                                   |                                                                            |                                                                                                                             | Reading magazines such as "Plus" published by The Mathematical Association.                     |
| Self-direction                                           | Planning and carrying out mathematical-based problem-solving under own direction.                                          | N/A                                                                        | N/A                                                                                                                         | Yes                                                                                             |
| Responsibility                                           | Taking responsibility for any errors or omissions in own work and creating a plan to improve.                              | e.g. 1.8D is about estimating answers                                      | e.g. 1F Qu 11<br>Candidate may estimate answer as $\frac{50 \times 3}{10}$ before carrying out calculation on a calculator. | Yes<br>Teaching style can encourage candidates to ask if an answer is "reasonable" or estimate. |
| Perseverance                                             | Actively seeking new ways to continue and improve own learning despite setbacks.                                           | N/A                                                                        | N/A                                                                                                                         | Yes                                                                                             |
| Productivity                                             | Using mathematical strategies and problem solving skills fluently                                                          | Some of the longer questions that require several steps would assess this. |                                                                                                                             | Yes                                                                                             |
| Self-regulation (metacognition, forethought, reflection) | Developing and refining a strategy over time for solving a problem, reflecting on the success or otherwise of the strategy | N/A                                                                        | N/A                                                                                                                         | Yes                                                                                             |
| Ethics                                                   | Producing output with a specific moral purpose for which one is accountable.                                               | N/A                                                                        | N/A                                                                                                                         | Yes                                                                                             |
| Integrity                                                | Taking ownership for own work and willingly responds to questions and challenges.                                          | N/A                                                                        | N/A                                                                                                                         | Yes                                                                                             |

| Positive Core Self Evaluation                      |                                                       |     |     |     |
|----------------------------------------------------|-------------------------------------------------------|-----|-----|-----|
| Self-monitoring/self-evaluation/self-reinforcement | Planning and reviewing own work as a matter of habit. | N/A | N/A | Yes |

| Skill                             | Skill interpretation in this subject                                                                                                  | Example of where the skill is covered in content | Where the skill is explicitly assessed in examination | Opportunity for the skill to be assessed formatively |
|-----------------------------------|---------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------|-------------------------------------------------------|------------------------------------------------------|
| <b>Interpersonal skills</b>       |                                                                                                                                       |                                                  |                                                       |                                                      |
| <b>Teamwork and collaboration</b> |                                                                                                                                       |                                                  |                                                       |                                                      |
| Communication                     | Able to communicate a mathematical process or technique (verbally or written) to peers and teachers and answer questions from others. | N/A                                              | N/A                                                   | Yes<br>e.g. in group discussion                      |
| Collaboration                     | Carrying out a peer review to provide supportive feedback to another.                                                                 | N/A                                              | N/A                                                   | Yes                                                  |
| Teamwork                          | Working with other students in a maths-based problem solving exercise.                                                                | N/A                                              | N/A                                                   | Yes                                                  |
| Co-operation                      | Sharing own resources and own learning techniques with other students.                                                                | N/A                                              | N/A                                                   | Yes                                                  |
| Interpersonal skills              | Using verbal and non-verbal communication skills in a dialogue about mathematics.                                                     | N/A                                              | N/A                                                   | Yes                                                  |

| Leadership              |                                                                                                                 |     |     |     |
|-------------------------|-----------------------------------------------------------------------------------------------------------------|-----|-----|-----|
| Leadership              | Leading others in a group activity to effectively solve a mathematical problem                                  | N/A | N/A | Yes |
| Responsibility          | Taking responsibility for the outcomes of a team exercise even if one is not solely responsible for the output. | N/A | N/A | Yes |
| Assertive communication | Chairing a debate, allowing representations and directing the conversation to a conclusion.                     | N/A | N/A | Yes |
| Self-presentation       | Presenting a mathematical problem to an audience to seek solutions.                                             | N/A | N/A | Yes |

## Appendix 2: Mapping grid for transferable skills for Maths B

| Skill                              | Skill interpretation in this subject                                                                                                                                                     | Example of where the skill is covered in content                                                                                            | Where the skill is explicitly assessed in examination                                            | Opportunity for the skill to be assessed formatively |
|------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------|------------------------------------------------------|
| <b>Cognitive skills</b>            |                                                                                                                                                                                          |                                                                                                                                             |                                                                                                  |                                                      |
| Cognitive Processes and Strategies |                                                                                                                                                                                          |                                                                                                                                             |                                                                                                  |                                                      |
| Critical thinking                  | Using <b>many</b> different pieces of mathematical information (sometimes seemingly unrelated) and synthesising this information to arrive at a solution to a mathematics-based problem. | e.g. 3I (quadratic and linear simultaneous equations)<br>9B (2D and 3D Trig)                                                                | Many of the longer questions on paper 2 address this e.g. Qu 7 (9B), Qu 8 (section 4). Qu 5 (3I) | Yes                                                  |
| Problem solving                    | Translating problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes and solve them.                                                   | See note in 1K but examples occur over a range of topics                                                                                    | e.g. Paper 2 Qu 1 (section 1)<br>Paper 1 Qu 21 (sections 1 & 3)                                  | Yes                                                  |
| Analysis                           | Examining and understanding different elements of a mathematical context or different mathematical processes.                                                                            | e.g. Curves, sketching and using calculus (4 J, K, L, M, N)<br>Links to solutions of equations (3G)                                         | e.g. Paper 2 Qu 8<br>Paper 1 Qu 27 (Quadratics and inequalities)<br>Qu 25 (section 4 N)          | Yes                                                  |
| Reasoning                          | Making abstract deductions and draw conclusions from mathematical information.                                                                                                           | Examples in Geometry involving congruence (6 I, J), similarity (6 H) and circle theorems (6K, L).<br>Also in vectors (8I) and matrices (5F) | e.g. Paper 1 Qu 17 (6I, J)<br>Qu 24 (6K)<br>Paper 2 Qu 11(c and d) (6H)<br>Qu 10 (f and g) (5F)  | Yes                                                  |

|                    |                                                                                                                                                 |                                                                                             |                                                                        |                                                                    |
|--------------------|-------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------|------------------------------------------------------------------------|--------------------------------------------------------------------|
| Interpretation     | Analysing mathematical information and understanding the meaning of that information, for example interpreting straight line conversion graphs. | Examples in many areas<br>e.g. 1G, 4N and Venn diagrams and probability (sections 2 and 10) | Paper 2 Qu 6 (4N)<br>Qu 3 (Sections 2 and 10)<br>Paper 1 Qu 25 (4N)    | Yes                                                                |
| Decision Making    | Selecting a mathematical process from a series of mathematical processes to solve a problem.                                                    | Examples in trigonometry (9B) and Pythagoras' theorem (6F)                                  | e.g. Paper 1 Qu 28 (9B)<br>Paper 2 Qu 7 (9B & 6F)                      | e.g. Use of discussion in whole class contexts or in small groups. |
| Adaptive learning  | Adapting a mathematical strategy to solve a context based mathematical problem.                                                                 | Examples in percentages and proportions (1 H) and probability (section 10)                  | e.g. Paper 2 Qu 1 (1H) Qu 9 (section 10)<br>Paper 1 Qu 14 (section 10) | Yes                                                                |
| Executive function | Planning how to solve a problem, carrying out the plan and reviewing the outcome.                                                               | Many longer, unstructured questions                                                         | e.g. Paper 1 Qu 17 (6I, Paper 2 Qu 11(d)<br>Qu 5 (3I)                  | Yes                                                                |

| Creativity |                                                                                                                                                                                                                |                                                                                                                                                                               |                                                                                                                                                                                                                                                                                                                                                                |                                                      |
|------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------|
| Creativity | <p>Using own learning to apply mathematical processes and link these together to prove and validate mathematical concepts</p> <p>Uses a different, unexpected mathematical process to arrive at an answer.</p> | <p>We use "Show that" style of questions where candidates have to give something approaching a proof. Sometimes we use "prove" in the context of congruent triangles (6J)</p> | <p>e.g. Paper 1 Qu 17 (6J)</p> <p>Qu 21 (number and algebra)</p> <p>Qu 26 (factor theorem 3D)</p> <p>Paper 2 Qu 4 (4M)</p> <p>Qu 11b (Vectors 8I)</p>                                                                                                                                                                                                          | <p>Yes</p> <p>May be evidenced in homework tasks</p> |
| Innovation | <p>Using a novel strategy to solve a previously unseen mathematical problem.</p>                                                                                                                               | <p>There is scope here in the area of turning points on curves (4M)</p>                                                                                                       | <p>Hard to explicitly assess but candidates may produce solutions not on mark scheme. e.g. to find the x-coordinate of the minimum on <math>y = 3x^2 - 9x + 5</math> the candidate uses ideas of symmetry and the mid-point of the roots. They may then use a knowledge that the sum of the roots is <math>-\frac{b}{a}</math> to write down the answer as</p> | <p>Yes</p> <p>See example.</p>                       |

|  |  |  |                                                                               |  |
|--|--|--|-------------------------------------------------------------------------------|--|
|  |  |  | $\frac{1}{2} \times \frac{9}{3} = \frac{3}{2}$ rather<br>than using calculus. |  |
|--|--|--|-------------------------------------------------------------------------------|--|

| Skill                               | Skill interpretation in this subject                                                                                                                        | Where the skill is covered in content                           | Where the skill is explicitly assessed in examination                                                                                                         | Opportunity for the skill to be assessed formatively                                                                                                        |
|-------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Intrapersonal skills</b>         |                                                                                                                                                             |                                                                 |                                                                                                                                                               |                                                                                                                                                             |
| Intellectual openness               |                                                                                                                                                             |                                                                 |                                                                                                                                                               |                                                                                                                                                             |
| Adaptability                        | Ability to select and apply knowledge and understanding of mathematical processes (that which is not prompted or provided) to unseen mathematical problems. | Many questions would assess this.                               | Yes<br>Any question where we do not specify the method to use e.g. Paper 1 Qus 10 (1D), 12 (6G), 15 (4H, I), 18 (3H), 20(6K, L)<br>Paper 2 Qus 2, 5 (Algebra) | Yes                                                                                                                                                         |
| Personal and social responsibility  | Using mathematical knowledge and skills to solve a problem for which one is accountable.                                                                    | Section 1K has a note all about applying number in everyday use | e.g. Paper 2 Qu 1 (1H)                                                                                                                                        | Yes<br>e.g. students monitoring their allowance                                                                                                             |
| Continuous learning                 | Planning and reflecting on own learning- setting goals and meeting them regularly                                                                           | N/A                                                             | N/A                                                                                                                                                           | Yes<br>Students identify areas where they need extra help or practice.                                                                                      |
| Intellectual interest and curiosity | Identifying a problem under own initiative, planning a solution and carrying this out.                                                                      | e.g. the topic of sequences lends itself to this (3B and 3L)    | e.g. Paper 1 Qu 5 <u>or</u> could give sequence 2, 8, 18, 32... and ask for the next two terms                                                                | Yes<br>Student goes on to try and find a formula for the $n$ th term ( $= 2n^2$ ) Not on specification but a simple question student could ask and explore. |

| Work ethic/<br>conscientiousness                                  |                                                                                                                            |                                                                                                                       |                                                                                                                                                         |                                                                                                 |
|-------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------|
| Initiative                                                        | Using mathematical knowledge, independently (without guided learning), to further own understanding.                       | N/A                                                                                                                   | N/A                                                                                                                                                     | Yes<br>Reading magazines such as "Plus" published by The Mathematical Association.              |
| Self-direction                                                    | Planning and carrying out mathematical-based problem-solving under own direction.                                          | N/A                                                                                                                   | N/A                                                                                                                                                     | Yes                                                                                             |
| Responsibility                                                    | Taking responsibility for any errors or omissions in own work and creating a plan to improve.                              | Section 1I covers rounding. Candidates can be encouraged to round to 1sf before evaluating an answer on a calculator. | e.g. Candidate may estimate answer as $\frac{50 \times 3}{10}$ before carrying out calculation of $\frac{51.2 \times 2.96}{3.1 + 7.4}$ on a calculator. | Yes<br>Teaching style can encourage candidates to ask if an answer is "reasonable" or estimate. |
| Perseverance                                                      | Actively seeking new ways to continue and improve own learning despite setbacks.                                           | N/A                                                                                                                   | N/A                                                                                                                                                     | Yes                                                                                             |
| Productivity                                                      | Using mathematical strategies and problem solving skills fluently (?)                                                      | Some of the longer questions that require several steps would assess this.                                            |                                                                                                                                                         | Yes                                                                                             |
| Self-regulation<br>(metacognition,<br>forethought,<br>reflection) | Developing and refining a strategy over time for solving a problem, reflecting on the success or otherwise of the strategy | N/A                                                                                                                   | N/A                                                                                                                                                     | Yes                                                                                             |
| Ethics                                                            | Producing output with a specific moral purpose for which one is accountable.                                               | N/A                                                                                                                   | N/A                                                                                                                                                     | Yes                                                                                             |

|                                                    |                                                                                   |     |     |     |
|----------------------------------------------------|-----------------------------------------------------------------------------------|-----|-----|-----|
| Integrity                                          | Taking ownership for own work and willingly responds to questions and challenges. | N/A | N/A | Yes |
| Positive Core Self Evaluation                      |                                                                                   |     |     |     |
| Self-monitoring/self-evaluation/self-reinforcement | Planning and reviewing own work as a matter of habit.                             | N/A | N/A | Yes |

### Appendix 3: Mapping grid for transferable skills for Further Pure Mathematics

| Skill                              | Skill interpretation in this subject                                                                                                                                                     | Example of where the skill is covered in content                                        | Where the skill is explicitly assessed in examination                                                      | Opportunity for the skill to be assessed formatively |
|------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------|------------------------------------------------------|
| <b>Cognitive skills</b>            |                                                                                                                                                                                          |                                                                                         |                                                                                                            |                                                      |
| Cognitive Processes and Strategies |                                                                                                                                                                                          |                                                                                         |                                                                                                            |                                                      |
| Critical thinking                  | Using <b>many</b> different pieces of mathematical information (sometimes seemingly unrelated) and synthesising this information to arrive at a solution to a mathematics-based problem. | e.g. 10 C (3D trig and Pythagoras)<br>10 H (trig equations)<br>2 C (roots of equations) | Paper 1 Qu 3 (10 H)<br>Qu 5 (algebra and calculus)<br>Qu 10 (10C), Qu 9 (2C)<br>Paper 2 Qu 4(9A, 3A, B, C) | Yes                                                  |
| Problem solving                    | Translating problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes and solve them.                                                   | Most topics have some application here.                                                 | e.g. Paper 1 Qu 11(a) (9A)<br>Paper 2 Qu 11 (9A, G, 10 C)                                                  | Yes                                                  |
| Analysis                           | Examining and understanding different elements of a mathematical context or different mathematical processes.                                                                            | e.g. (study of shape of graphs, turning points, roots etc 4A, B and 9 D, E)             | e.g. Paper 1 Qu 11                                                                                         | Yes                                                  |

|                    |                                                                                                                                                 |                                                                                                                              |                                                                           |                                                                    |
|--------------------|-------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------|--------------------------------------------------------------------|
| Reasoning          | Making abstract deductions and draw conclusions from mathematical information.                                                                  | e.g. Use of discriminant (2B ) and roots of equations (9C) and vectors (9F)                                                  | Paper 2 Qu 9 (2A)<br>Paper 1 Qu 9 (9C)                                    | Yes                                                                |
| Interpretation     | Analysing mathematical information and understanding the meaning of that information, for example interpreting straight line conversion graphs. | Most topics cover this.<br>e.g. 9C (Kinematics)<br>3 E (Inequalities and linear programming)                                 | e.g. Paper 1 Qu 4 (9C)<br><br>Qu 1 (3 E)                                  | Yes                                                                |
| Decision Making    | Selecting a mathematical process from a series of mathematical processes to solve a problem.                                                    | e.g. Selection of appropriate method in Trig and Pythagoras problems (10 C)<br>Use of rules of logarithms (1 B)              | e.g. Paper 1 Qu 10 (10 C)<br><br>Paper 2 Qu 6 (1B)                        | e.g. Use of discussion in whole class contexts or in small groups. |
| Adaptive learning  | Adapting a mathematical strategy to solve a context based mathematical problem.                                                                 | Not many examples here as the focus is on Pure Mathematics. An example linking 3C and 9E                                     | e.g. Paper 1 Qu 5 (3C & 9E)                                               | Yes                                                                |
| Executive function | Planning how to solve a problem, carrying out the plan and reviewing the outcome.                                                               | Questions in calculus and connected rates of change (9 G) require candidates to select the appropriate stages (i.e. “plan”?) | e.g. Paper 2 Qu 11 (9 G)                                                  | Yes                                                                |
| Creativity         |                                                                                                                                                 |                                                                                                                              |                                                                           |                                                                    |
| Creativity         | Using own learning to apply mathematical processes and link these together to prove and validate mathematical concepts                          | We use “Show that” style of questions where candidates have to give something approaching a proof                            | e.g. Paper 2 Qu 5(a) (10 G)<br><br>Paper 1 Qu 5(a) (3 C)<br>Qu 9(b) (2 C) | Yes<br>May be evidenced in homework tasks                          |

|            |                                                                           |                                                                                                      |                                                                                                                                                                                                                                                                                                                                                                                                                 |                     |
|------------|---------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------|
|            | Uses a different, unexpected mathematical process to arrive at an answer. | e.g. with trigonometric formulae (10 G)<br><br>Also 7 F requires simple ideas of proof using vectors | Qu 10(d)<br>(trigonometry and surds)                                                                                                                                                                                                                                                                                                                                                                            |                     |
| Innovation | Using a novel strategy to solve a previously unseen mathematical problem. | There is scope here in the area of turning points on curves (sections 9 D and 9 E and links to 2 C)  | Hard to explicitly assess but candidates may produce solutions not on mark scheme.<br>e.g. to find the x-coordinate of the minimum on $y = 3x^2 - 9x + 5$ the candidate uses ideas of symmetry and the mid-point of the roots. They may then use a knowledge that the sum of the roots is $-\frac{b}{a}$ to write down the answer as $\frac{1}{2} \times \frac{9}{3} = \frac{3}{2}$ rather than using calculus. | Yes<br>See example. |

| Skill                               | Skill interpretation in this subject                                                                                                                        | Example of where the skill is covered in content                                                                                                               | Where the skill is explicitly assessed in examination                               | Opportunity for the skill to be assessed formatively                         |
|-------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|------------------------------------------------------------------------------|
| <b>Intrapersonal skills</b>         |                                                                                                                                                             |                                                                                                                                                                |                                                                                     |                                                                              |
| Intellectual openness               |                                                                                                                                                             |                                                                                                                                                                |                                                                                     |                                                                              |
| Adaptability                        | Ability to select and apply knowledge and understanding of mathematical processes (that which is not prompted or provided) to unseen mathematical problems. | Many questions would assess this                                                                                                                               | Yes<br>Any question where we do not specify the method to use<br>e.g. Paper 2 Qu 11 | Yes                                                                          |
| Personal and social responsibility  | Using mathematical knowledge and skills to solve a problem for which one is accountable.                                                                    | The spec. is based on just Pure mathematics however:<br>3 E has the option of simple linear programming questions<br>9 C relates calculus to simple kinematics |                                                                                     | Yes<br>e.g. students organise a disco and need to determine price of tickets |
| Continuous learning                 | Planning and reflecting on own learning- setting goals and meeting them regularly                                                                           | N/A                                                                                                                                                            | N/A                                                                                 | Yes<br>Students identify areas where they need extra help or practice.       |
| Intellectual interest and curiosity | Identifying a problem under own initiative, planning a solution and carrying this out.                                                                      | e.g. 4 B<br>Student uses graphical techniques to solve more complex equations.                                                                                 | e.g. Paper 1 Qu 7                                                                   | Yes<br>e.g. student tries to solve $\log_2(4x-6)^2 - x^2 = 2$                |
| Work ethic/conscientiousness        |                                                                                                                                                             |                                                                                                                                                                |                                                                                     |                                                                              |
| Initiative                          | Using mathematical knowledge, independently (without guided                                                                                                 | N/A                                                                                                                                                            | N/A                                                                                 | Yes                                                                          |

|                                                          |                                                                                                                            |                                                                            |                                                                                                                                                                                                                                              |                                                                                                 |
|----------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------|
|                                                          | learning), to further own understanding.                                                                                   |                                                                            |                                                                                                                                                                                                                                              | Reading magazines such as “Plus” published by The Mathematical Association.                     |
| Self-direction                                           | Planning and carrying out mathematical-based problem-solving under own direction.                                          | N/A                                                                        | N/A                                                                                                                                                                                                                                          | Yes                                                                                             |
| Responsibility                                           | Taking responsibility for any errors or omissions in own work and creating a plan to improve.                              | e.g. using a calculator to check answers                                   | e.g. Paper 2 Qu 8 candidate uses a calculator to compare their final approximation with calculator value.<br>Paper 1 Qu 11(b) candidate uses a calculator (or approx.. area of triangle as $0.5 \times 4 \times 30$ ) to check final answer. | Yes<br>Teaching style can encourage candidates to ask if an answer is “reasonable” or estimate. |
| Perseverance                                             | Actively seeking new ways to continue and improve own learning despite setbacks.                                           | N/A                                                                        | N/A                                                                                                                                                                                                                                          | Yes                                                                                             |
| Productivity                                             | Using mathematical strategies and problem solving skills fluently (?)                                                      | Some of the longer questions that require several steps would assess this. |                                                                                                                                                                                                                                              | Yes                                                                                             |
| Self-regulation (metacognition, forethought, reflection) | Developing and refining a strategy over time for solving a problem, reflecting on the success or otherwise of the strategy | N/A                                                                        | N/A                                                                                                                                                                                                                                          | Yes                                                                                             |
| Ethics                                                   | Producing output with a specific moral purpose for which one is accountable.                                               | N/A                                                                        | N/A                                                                                                                                                                                                                                          | Yes                                                                                             |

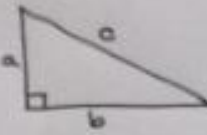
|                                                    |                                                                                   |     |     |     |
|----------------------------------------------------|-----------------------------------------------------------------------------------|-----|-----|-----|
| Integrity                                          | Taking ownership for own work and willingly responds to questions and challenges. | N/A | N/A | Yes |
| Positive Core Self Evaluation                      |                                                                                   |     |     |     |
| Self-monitoring/self-evaluation/self-reinforcement | Planning and reviewing own work as a matter of habit.                             | N/A | N/A | Yes |

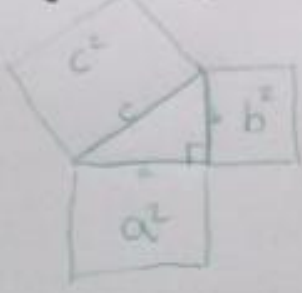
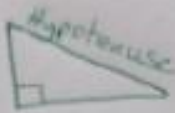
| Skill                       | Skill interpretation in this subject                                                                                                  | Example of where the skill is covered in content | Where the skill is explicitly assessed in examination | Opportunity for the skill to be assessed formatively |
|-----------------------------|---------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------|-------------------------------------------------------|------------------------------------------------------|
| <b>Interpersonal skills</b> |                                                                                                                                       |                                                  |                                                       |                                                      |
| Teamwork and collaboration  |                                                                                                                                       |                                                  |                                                       |                                                      |
| Communication               | Able to communicate a mathematical process or technique (verbally or written) to peers and teachers and answer questions from others. | N/A                                              | N/A                                                   | Yes<br>e.g. in group discussion                      |
| Collaboration               | Carrying out a peer review to provide supportive feedback to another.                                                                 | N/A                                              | N/A                                                   | Yes                                                  |
| Teamwork                    | Working with other students in a maths-based problem solving exercise.                                                                | N/A                                              | N/A                                                   | Yes                                                  |
| Co-operation                | Sharing own resources and own learning techniques with other students.                                                                | N/A                                              | N/A                                                   | Yes                                                  |
| Interpersonal skills        | Using verbal and non-verbal communication skills in a dialogue about mathematics.                                                     | N/A                                              | N/A                                                   | Yes                                                  |

| Leadership              |                                                                                                                 |     |     |     |
|-------------------------|-----------------------------------------------------------------------------------------------------------------|-----|-----|-----|
| Leadership              | Leading others in a group activity to effectively solve a mathematical problem                                  | N/A | N/A | Yes |
| Responsibility          | Taking responsibility for the outcomes of a team exercise even if one is not solely responsible for the output. | N/A | N/A | Yes |
| Assertive communication | Chairing a debate, allowing representations and directing the conversation to a conclusion.                     | N/A | N/A | Yes |
| Self-presentation       | Presenting a mathematical problem to an audience to seek solutions.                                             | N/A | N/A | Yes |

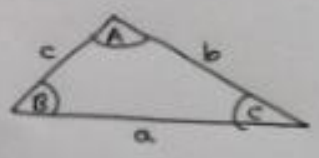
# Appendix 4: 'Written relay' on Pythagoras' theorem and trigonometric facts

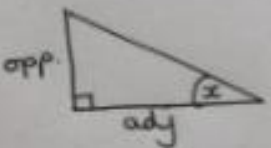
Pythagoras' theorem and Trigonometry

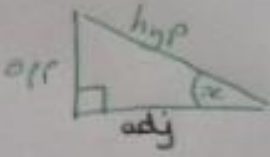


$$a^2 + b^2 = c^2$$



Hypotenuse



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$


$$\tan x = \frac{\text{opp}}{\text{adj}}$$


$$\sin x = \frac{\text{opp}}{\text{hyp}}$$

$$\cos x = \frac{\text{adj}}{\text{hyp}}$$

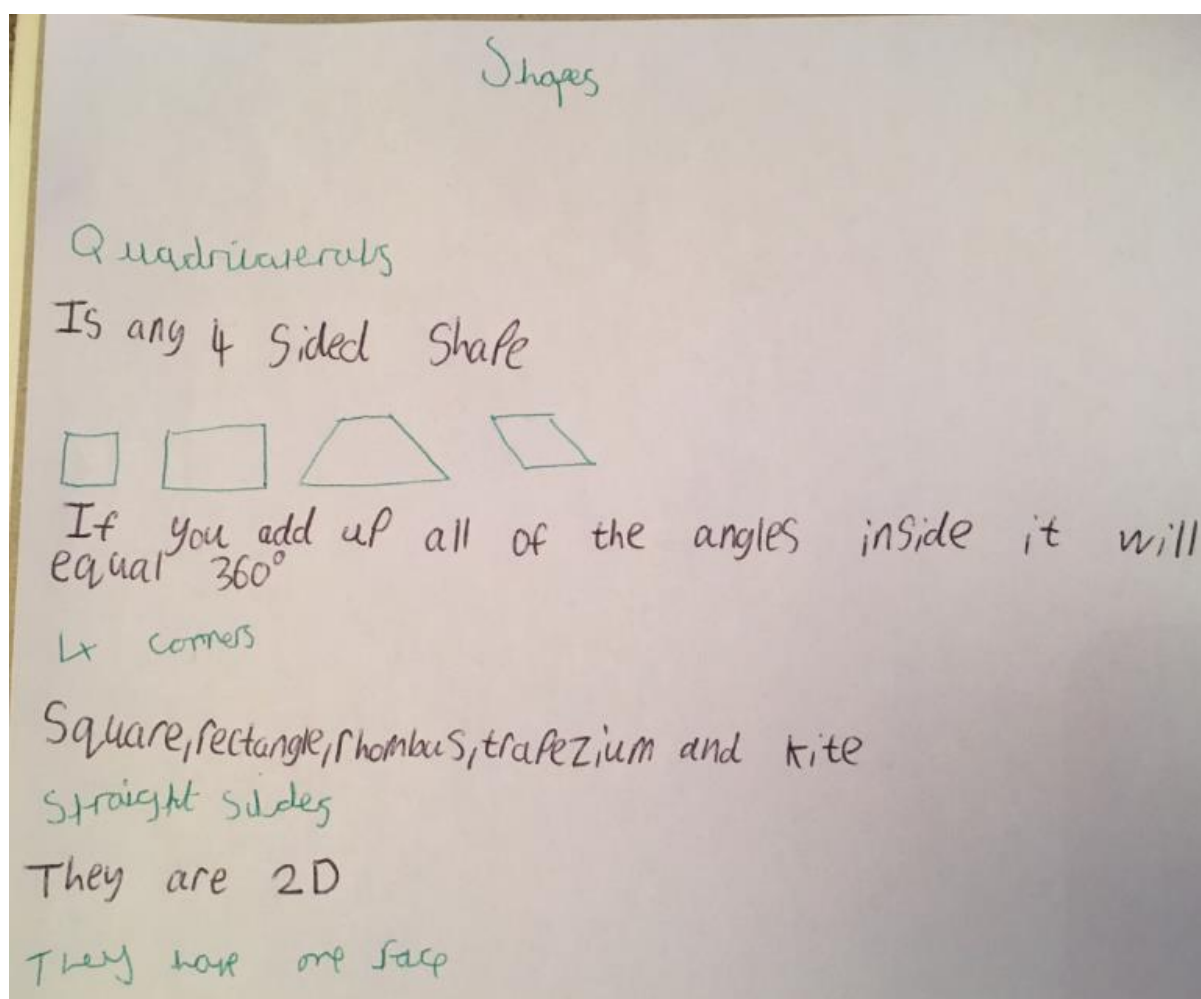

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\text{Area} = \frac{1}{2} bc \sin A$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{Area} = \frac{1}{2} ab \sin C$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$



## Appendix 6: Problem-solving with Pythagoras' theorem and trigonometry lesson plan

|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |                                                                     |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------|
| <b>Objective</b> <ul style="list-style-type: none"> <li>Use Pythagoras' theorem and trigonometry to solve problems.</li> </ul>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | <b>Resources</b><br>Pythagoras' theorem and trigonometry worksheet. |
| <b>Transferable skills</b> <ul style="list-style-type: none"> <li><b>Critical thinking</b> (Using many different pieces of mathematical information (sometimes seemingly unrelated) and synthesising this information to arrive at a solution to a mathematics-based problem.)</li> <li><b>Problem solving</b> (Translating problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes and solve them.)</li> <li><b>Decision making</b> (Selecting a mathematical process from a series of mathematical processes to solve a problem.)</li> <li><b>Adaptability</b> (Ability to select and apply knowledge and understanding of mathematical processes (that which is not prompted or provided) to unseen mathematical problems.)</li> <li><b>Communication</b> (Able to communicate a mathematical process or technique (verbally or written) to peers and teachers and answer questions from others.)</li> <li><b>Collaboration</b> (Carrying out a peer review to provide supportive feedback to another.)</li> <li><b>Co-operation</b> (Sharing own resources and own learning techniques with other students.)</li> <li><b>Interpersonal skills</b> (Using verbal and non-verbal communication skills in a dialogue about mathematics.)</li> </ul> |                                                                     |

### Starter

Students work in a group using a 'Structured group' to decide if each triangle with the following dimensions is a right-angled triangle or not. Students work in a group of 4 to solve the task and they number themselves 1 to 4. When all groups have their solutions and explanations ready the teacher chooses one number and the student in each group with that number stands. That student is then the spokesperson for the group and feeds any results back to the teacher and the class (therefore, it is important that every student in each group understands the solutions as they don't know which student will be the spokesperson until the task is complete). (**Problem solving**, **Decision making**, **Adaptability**, **Communication**, **Collaboration**, **Co-operation**, **Interpersonal skills**)

- Triangle 1: 6cm, 8cm, 10cm
- Triangle 2: 4cm, 7cm, 8cm
- Triangle 3: 5cm, 12cm, 13cm
- Triangle 4: 9cm, 40cm, 41cm
- Triangle 5: 12cm, 13cm, 18cm
- Triangle 6: 65cm, 72cm, 97cm

## Main lesson

### Teaching points

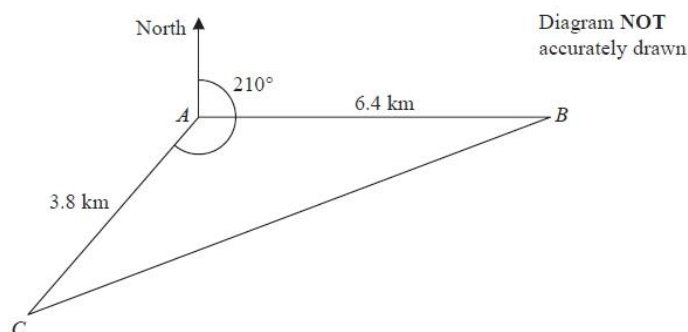
In this lesson students use Pythagoras' theorem and trigonometry to help them solve problems.

- Students complete a timed 'Written relay' on Pythagoras' theorem and trigonometry. Students work in pairs or a small group with one piece of paper. Students take turns to write something they know/a fact about Pythagoras' theorem or trigonometry (students must not repeat anything that is already on the paper so they must watch what their partner/group are writing). Once the time is up, one student from each pair/group chooses one fact from their paper to share with the class (teacher could choose the student to speak randomly by saying it is to be the student who has the largest handspan/the student whose birthday is the closest to 1st September/etc). Each group takes a turn to share a fact until all facts have been shared/are written on the board. (Communication, Co-operation, Interpersonal skills)
- Recap on Pythagoras' theorem and trigonometry by applying the notes students made from the previous activity to a couple of examples.
- Students complete the attached worksheet using 'Paired coaching'. Students work in pairs to answer the questions on the worksheet. Student A answers their question 1, whilst student B watches them, checking their working as they go. When student A has completed question 1 student B says whether they think the solution is right or not. If they believe the answer is incorrect then they coach student A through where they have gone wrong so that they can correct their solution if necessary. When the students both agree that the solution is correct student B answers their question 1, whilst student A watches them, checking their working as they go. When student B has completed question 1 student A says whether they think the solution is right or not. If they believe the answer is incorrect then they coach student B through where they have gone wrong so that they can correct their solution if necessary. Then repeat this process for the rest of the worksheet. (Critical thinking, Problem solving, Decision making, Adaptability, Communication, Co-operation, Interpersonal skills)

## Plenary

A candidate presents this solution to an exam question. Ask students to work in pairs to analyse the solution and explain where the candidate has gone

wrong. First give students a given amount of time to attempt the task on their own and then they share their solution with their partner and must agree on a solution before presenting it to the class. (Communication, Collaboration, Co-operation, Interpersonal skills)



A, B and C are 3 villages.

B is 6.4 km due east of A.

C is 3.8 km from A on a bearing of  $210^\circ$

Calculate the bearing of B from C.

Give your answer correct to the nearest degree.

Show your working clearly.

(6 marks)

Using the cosine rule  $a^2 = b^2 + c^2 - 2bc \cos A$

$$BC^2 = 3.8^2 + 6.4^2 - 2 \times 3.8 \times 6.4 \times \cos 210^\circ$$

$$= 97.52347...$$

$$BC = 9.875397...$$

Using the cosine rule to find angle B

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$= \frac{9.875397^2 + 6.4^2 - 3.8^2}{2 \times 9.875397 \times 6.4}$$

$$= 0.981317...$$

$$B = 11.1^\circ$$

$$\text{Bearing} = 270 - 11.1 = 259^\circ \text{ (to the nearest deg)}$$

The two mistakes in the candidate's working are:

1. The candidate has used the angle of  $210^\circ$  to work out BC, it should be  $120^\circ$  (the interior angle of the triangle at A is  $210^\circ - 90^\circ$ )
2. The candidate has worked out the bearing of C from B and not B from C as asked.

## Appendix 7: 'Paired coaching' worksheets

$$\begin{array}{r} 1 \quad 46 \\ \underline{3 \times} \\ \hline \end{array}$$

$$\begin{array}{r} 2 \quad 57 \\ \underline{6 \times} \\ \hline \end{array}$$

$$\begin{array}{r} 3 \quad 81 \\ \underline{9 \times} \\ \hline \end{array}$$

$$\begin{array}{r} 4 \quad 42 \\ \underline{5 \times} \\ \hline \end{array}$$

$$\begin{array}{r} 5 \quad 83 \\ \underline{7 \times} \\ \hline \end{array}$$

$$\begin{array}{r} 6 \quad 37 \\ \underline{4 \times} \\ \hline \end{array}$$

$$\begin{array}{r} 7 \quad 286 \\ \underline{5 \times} \\ \hline \end{array}$$

$$\begin{array}{r} 8 \quad 562 \\ \underline{3 \times} \\ \hline \end{array}$$

$$\begin{array}{r} 9 \quad 193 \\ \underline{7 \times} \\ \hline \end{array}$$

$$\begin{array}{r} 10 \quad 601 \\ \underline{8 \times} \\ \hline \end{array}$$

$$\begin{array}{r} 11 \quad 734 \\ \underline{2 \times} \\ \hline \end{array}$$

$$\begin{array}{r} 12 \quad 921 \\ \underline{6 \times} \\ \hline \end{array}$$

$$\begin{array}{r} 1 \quad 38 \\ \underline{3 \times} \\ \hline \end{array}$$

$$\begin{array}{r} 2 \quad 52 \\ \underline{8 \times} \\ \hline \end{array}$$

$$\begin{array}{r} 3 \quad 76 \\ \underline{9 \times} \\ \hline \end{array}$$

$$\begin{array}{r} 4 \quad 54 \\ \underline{6 \times} \\ \hline \end{array}$$

$$\begin{array}{r} 5 \quad 76 \\ \underline{5 \times} \\ \hline \end{array}$$

$$\begin{array}{r} 6 \quad 92 \\ \underline{7 \times} \\ \hline \end{array}$$

$$\begin{array}{r} 7 \quad 247 \\ \underline{3 \times} \\ \hline \end{array}$$

$$\begin{array}{r} 8 \quad 467 \\ \underline{7 \times} \\ \hline \end{array}$$

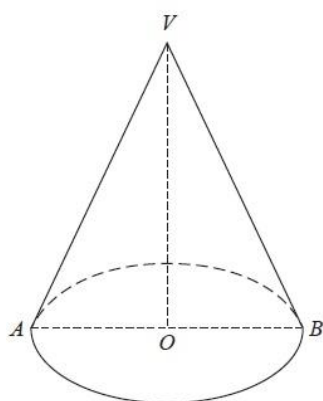
$$\begin{array}{r} 9 \quad 189 \\ \underline{5 \times} \\ \hline \end{array}$$

$$\begin{array}{r} 10 \quad 607 \\ \underline{6 \times} \\ \hline \end{array}$$

$$\begin{array}{r} 11 \quad 852 \\ \underline{8 \times} \\ \hline \end{array}$$

$$\begin{array}{r} 12 \quad 943 \\ \underline{2 \times} \\ \hline \end{array}$$

1.

Diagram NOT  
accurately drawn

The diagram shows a solid cone.  
The base of the cone is a horizontal circle,  
centre  $O$ , with radius 4.5 cm.

$AB$  is a diameter of the base and  $OV$  is the  
vertical height of the cone.

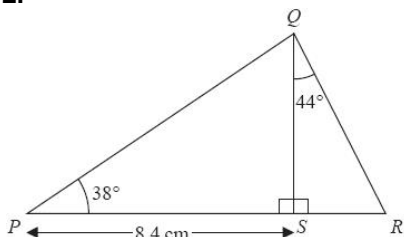
The curved surface area of the cone is  $130\text{ cm}^2$

Calculate the size of the angle  $AVB$ .

Give your answer correct to 1 decimal place.

(4 marks)

2.

Diagram NOT  
accurately drawn

$PSR$  is a straight line.

Angle  $PSQ = 90^\circ$

$PS = 8.4\text{ cm}$

Angle  $QPS = 38^\circ$

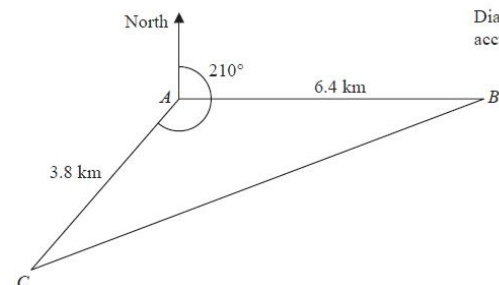
Angle  $SQR = 44^\circ$

Work out the length of  $QR$ .

Give your answer correct to 3 significant  
figures.

(4 marks)

3.

Diagram NOT  
accurately drawn

$A$ ,  $B$  and  $C$  are 3 villages.

$B$  is 6.4 km due east of  $A$ .

$C$  is 3.8 km from  $A$  on a bearing of  $210^\circ$

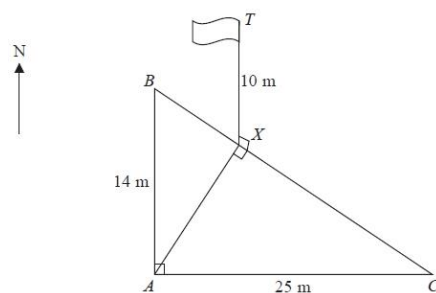
Calculate the bearing of  $B$  from  $C$ .

Give your answer correct to the nearest  
degree.

Show your working clearly.

(6 marks)

1.

Diagram NOT  
accurately drawn

$A$ ,  $B$  and  $C$  are points on horizontal ground.

$B$  is due North of  $A$  and  $AB$  is 14 m.

$C$  is due East of  $A$  and  $AC$  is 25 m.

A vertical flagpole,  $TX$ , has its base at the  
point  $X$  on  $BC$  such that the angle  $AXC$  is a  
right angle.

The height of the flagpole,  $TX$ , is 10 m.

Calculate the size of the angle of elevation of  $T$   
from  $A$ .

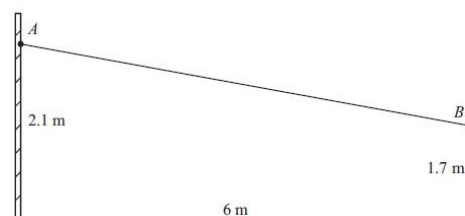
Give your answer correct to 1 decimal place.

(6 marks)

2. A washing line is attached at points  $A$   
and  $B$  on two vertical posts standing on  
horizontal ground.

Point  $A$  is 2.1 metres above the ground on one  
post.

Point  $B$  is 1.7 metres above the ground on the  
other post. The horizontal distance between  
the two posts is 6 metres.

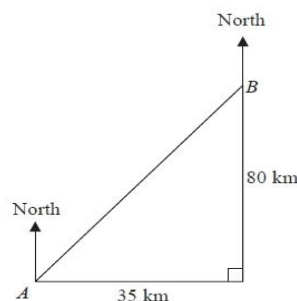
Diagram NOT  
accurately drawn

Calculate the distance  $AB$ .

Give your answer correct to 3 significant  
figures.

(4 marks)

3.

Diagram NOT  
accurately drawn

Town  $B$  is 35 km east and 80 km north of town

$A$ . Work out the bearing of town  $A$  from town

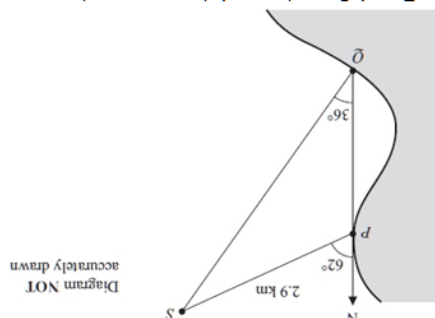
$B$ . Give your answer correct to the nearest  $d$   
degree.

(4 marks)

## Appendix 8: 'Question roundabout' using Pythagoras' theorem and trigonometry past exam questions

(3 marks)  
Give your answer correct to 3 significant figures.

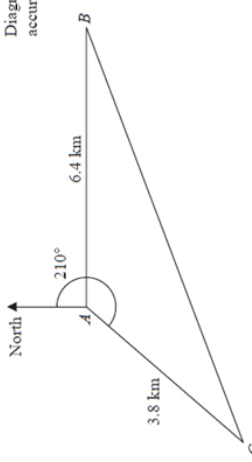
Calculate the distance  $QS$ .  
The bearing of the ship from  $P$  is  $062^\circ$ .  
 $PS = 2.9$  km.  
A ship is at the point  $S$ .  
 $P$  is due North of  $Q$ .  
 $P$  and  $Q$  are two points on a coast.



1.

2.

Diagram NOT accurately drawn



$A$ ,  $B$  and  $C$  are 3 villages.

$B$  is 6.4 km due east of  $A$ .

$C$  is 3.8 km from  $A$  on a bearing of  $210^\circ$ .

Calculate the bearing of  $B$  from  $C$ .

Give your answer correct to the nearest degree.

Show your working clearly.

(6 marks)

4. A circular clock face, centre  $O$ , has a minute hand  $OA$  and an hour hand  $OB$ .

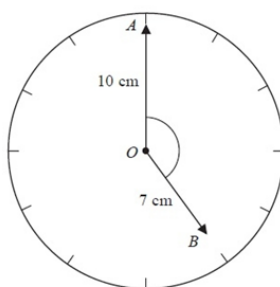
$OA = 10$  cm.

$OB = 7$  cm.

Calculate the length of  $AB$  when the hands show 5 o'clock.

Give your answer correct to 3 significant figures.

Diagram NOT accurately drawn



(4 marks)

| Objective                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | Resources              |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------|
| <ul style="list-style-type: none"> <li>• Use sequences to solve problems.</li> </ul><br><b>Transferable skills</b> <ul style="list-style-type: none"> <li>• <b>Critical thinking</b> (Using many different pieces of mathematical information (sometimes seemingly unrelated) and synthesising this information to arrive at a solution to a mathematics-based problem.)</li> <li>• <b>Problem solving</b> (Translating problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes and solve them.)</li> <li>• <b>Decision making</b> (Selecting a mathematical process from a series of mathematical processes to solve a problem.)</li> <li>• <b>Adaptive learning</b> (Adapting a mathematical strategy to solve a context based mathematical problem.)</li> <li>• <b>Executive function</b> (Planning how to solve a problem, carrying out the plan and reviewing the outcome.)</li> <li>• <b>Adaptability</b> (Ability to select and apply knowledge and understanding of mathematical processes (that which is not prompted or provided) to unseen mathematical problems.)</li> <li>• <b>Personal and social responsibility</b> (Using mathematical knowledge and skills to solve a problem for which one is accountable.)</li> <li>• <b>Intellectual interest and curiosity</b> (Identifying a problem under own initiative, planning a solution and carrying this out.)</li> <li>• <b>Initiative</b> (Using mathematical knowledge, independently (without guided learning), to further own understanding.)</li> <li>• <b>Self-direction</b> (Planning and carrying out mathematical-based problem-solving under own direction.)</li> <li>• <b>Self-regulation</b> (Developing and refining a strategy over time for solving a problem, reflecting on the success or otherwise of the strategy)</li> <li>• <b>Self-monitoring/self-evaluation/self-reinforcement</b> (Planning and reviewing own work as a matter of habit.)</li> <li>• <b>Interpersonal skills</b> (Using verbal and non-verbal communication skills in a dialogue about mathematics.)</li> </ul> | Number grid worksheet. |

## Starter

Students work in a group of 4 to solve the task 'Handshakes'.

Ask students to work out how many handshakes will take place in their group of 4 if they all shake hands with each other.

Ask students to work out how many handshakes will take place if someone else joins their group.

Ask students to work out how many handshakes will take place if there are 10 people in the group.

Ask students to work out how many handshakes will take place if there are 30 people in the group.

Ask students to work out how many handshakes will take place if there are  $n$  people in the group.

Ask students to reflect how they developed the task from the number of handshakes for 4 and 5 people to 10, then 30 and then  $n$  people.

(Critical thinking, Problem solving, Decision making, Adaptive learning, Executive function, Adaptability, Personal and social responsibility, Intellectual interest and curiosity, Initiative, Self-direction, Self-regulation, Self-monitoring/self-evaluation/self-reinforcement, Interpersonal skills)

## Main lesson

### Teaching points

In this lesson students investigate the 'number grids problem' as shown on the attached worksheet:

Look at this number grid:

|    |    |    |    |    |    |    |    |    |     |
|----|----|----|----|----|----|----|----|----|-----|
| 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10  |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20  |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30  |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40  |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50  |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60  |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70  |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80  |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90  |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

A box is drawn around four numbers, find the product of the top left number and the bottom right number in this box.

Now do the same with the top right and bottom left numbers.

Finally calculate the difference between these products.

Investigate further.

## Main lesson

Give students thinking time, if they do not know where to start do not tell them what to do, instead, ask them generic questions that could be applied to any investigation/problem-solving task such as:

- Can you state the problem in your own words? (This checks that they understand what is being asked of them)
- What are you being asked to find out?
- What topics might this be to do with?
- What facts are given? How are you going to use them?
- What are you going to do first?

As students work through the task you could ask:

- How will you record what you're going to do?
- Are your results arranged in a logical order so that patterns are easier to spot?
- How will you know that you have done enough?

If this is the first investigation that students have done they may need more guidance such as trying the square in different places on the grid, trying different size squares on the grid, trying rectangles on the grid, trying different size grids, etc. As students gain more practice with investigations/open-ended tasks, they will grow in confidence and should need less and less guidance.

(Critical thinking, Problem solving, Decision making, Adaptive learning, Executive function, Adaptability, Personal and social responsibility, Intellectual interest and curiosity, Initiative, Self-direction, Self-regulation, Self-monitoring/self-evaluation/self-reinforcement, Interpersonal skills)

## Plenary

This plenary is designed to get students to reflect on the way they tackled the problem.

Ask students:

- How did you get on?
- What difficulties did you face when starting the task?
- Did anything stop you making progress?
- What questions did you ask yourselves?
- Did you change direction or approach at any time?
- What extra information did you need to be able to start the problem?
- What sort of hints would have helped you with this problem?

(Self-monitoring/self-evaluation/self-reinforcement, Interpersonal skills)

## Number Grid

Look at this number grid:

|    |    |    |    |    |    |    |    |    |     |
|----|----|----|----|----|----|----|----|----|-----|
| 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10  |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20  |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30  |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40  |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50  |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60  |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70  |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80  |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90  |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

A box is drawn around four numbers, find the product of the top left number and the bottom right number in this box.

Now do the same with the top right and bottom left numbers.

Finally calculate the difference between these products.

Investigate further.