

INTERNATIONAL ADVANCED LEVEL

MATHEMATICS/ FURTHER MATHEMATICS/ PURE MATHEMATICS

SCHEME OF WORK STATISTICS 1

Pearson Edexcel International Advanced Subsidiary in Mathematics (XMA01)

Pearson Edexcel International Advanced Subsidiary in Further Mathematics (XFM01)

Pearson Edexcel International Advanced Subsidiary in Pure Mathematics (XPM01)

Pearson Edexcel International Advanced Level in Mathematics (YMA01)

Pearson Edexcel International Advanced Level in Further Mathematics (YFM01)

Pearson Edexcel International Advanced Level in Pure Mathematics (YPM01)

First teaching September 2018

First examination from January 2019

First certification from August 2019 (International Advanced Subsidiary) and August 2020
(International Advanced Level)



Statistics 1

Unit	Title	Estimated hours
1	Representation and summary of data	
<u>a</u>	Calculation and interpretation of measures of location; Calculation and interpretation of measures of variation; Understand and use coding	5
<u>b</u>	Use statistical diagrams for single-variable data to draw simple conclusions and to compare distributions; Understand and identify outliers; Understand and determine skewness	8
2	Probability:	
<u>a</u>	Mutually exclusive events; Independent events	4
<u>b</u>	Using set notation for probability; Conditional probability	6
3	Correlation and regression	
<u>a</u>	Scatter diagrams and least squares linear regression	9
<u>b</u>	The product moment correlation coefficient	7
4	Discrete random variables	
<u>a</u>	Use a discrete probability distribution to model simple situations; Identify the discrete uniform distribution	6
<u>b</u>	Mean and variance of discrete probability distributions	7
5	The Normal distribution: Understand and use the Normal distribution	8
		60 hours

UNIT 1: Representation and summary of data[Return to overview](#)**SPECIFICATION REFERENCES**

- 2.1** Histograms, stem and leaf diagrams, box plots
- 2.2** Measures of location – mean, median, mode
- 2.3** Measures of dispersion – variance, standard deviation, range and interpercentile ranges
- 2.4** Skewness. Concepts of outliers

PRIOR KNOWLEDGEInternational GCSE/GCSE (9–1) in Mathematics at Higher Tier

- Interpret and construct tables, charts and diagrams, including frequency tables, bar charts, pie charts and pictograms for categorical data, vertical line charts for ungrouped discrete numerical data and know their appropriate use
- Construct and interpret diagrams for grouped discrete data and continuous data, i.e. histograms with equal and unequal class intervals and cumulative frequency graphs, and know their appropriate use
- Interpret, analyse and compare the distributions of data sets from univariate empirical distributions through appropriate measures of central tendency (median, mean, mode and modal class) and spread (range, including consideration of outliers), quartiles and inter-quartile range
- Use and interpret scatter graphs of bivariate data; recognise correlation and know that it does not indicate causation; draw estimated lines of best fit; make predictions; interpolate and extrapolate apparent trends while knowing the dangers of so doing (GCSE only)

KEYWORDS

Histogram, box plot, mean, median, mode, variance, standard deviation, range, interquartile range, interpercentile range, outlier, skewness, symmetrical, positive skew, negative skew.

1a. Calculation and interpretation of measures of location; Calculation and interpretation of measures of variation; Understand and use coding (2.2) (2.3)

Teaching time
5 hours

OBJECTIVES

By the end of the sub-unit, students should:

- be able to calculate measures of location, mean, median and mode;
- be able to calculate measures of variation, standard deviation, variance, range and interpercentile range;
- be able to interpret and draw inferences from summary statistics.

TEACHING POINTS

The calculation of the mean, median and mode should be recapped from International GCSE/GCSE, however the focus now is on students using calculators to do the calculations. Check understanding of the terminology and teach calculator methods.

Students require an understanding of measures of variation too and should be able to use their calculators to calculate the variance and standard deviation. They should be able to use the statistic

$$S_{xx} = \sum (x - \bar{x})^2 = \sum x^2 - \frac{(\sum x)^2}{n}. \text{ Students are expected to use standard deviation } = \sqrt{\frac{S_{xx}}{n}}, \text{ but}$$

equivalents including spreadsheet formula ($s = \sqrt{\frac{S_{xx}}{n-1}}$) will be accepted.

The data may be discrete or continuous, grouped or ungrouped, and students need to be able to interpret these summary statistics clearly and be able to make inferences from them. Significance tests will not be expected.

Coding for both mean and standard deviation needs to be covered. Be clear that students need to be able to uncode both mean and standard deviation. Emphasise that the standard deviation is unaffected by the addition or subtraction of constants.

Students are expected to be able to use linear interpolation to calculate the median and percentiles from grouped data.

OPPORTUNITIES FOR PROBLEM SOLVING/MODELLING

Summary statistics can be calculated from real-life data and then used to compare and interpret for both location and variation statistics.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

When calculating the mean of grouped data some students may divide by the number of groups rather than the number of items of data, they may also use class widths in the calculation rather than the mid-points.

When finding the standard deviation, the most common error is forgetting to take the square root (perhaps because they are not clear about the difference between variance and standard deviation). Some students waste time by ignoring given values and recalculating $\sum fx$ and $\sum fx^2$.

Difficulties with coding are due to a lack of understanding about how coding affects the mean and standard deviation, and poor algebraic skills. Students sometimes substitute for the wrong variable, fail to solve equations correctly or get the order of operations the wrong way around.

A level Further Mathematics: Statistics 1

Students should be reminded that they must be precise in their use of language and use the correct terms such as ‘median’, ‘range’ or ‘inter-quartile range’ rather than the more general ‘average’ and ‘spread’. Students should also remember to use accurate values throughout calculations to avoid losing marks due to premature rounding.

NOTES

Students are expected to know the different notation for population summary statistics (μ , σ^2 , σ) and sample summary statistics ($\bar{x}$, s^2 , s).

1b. Use statistical diagrams for single-variable data to draw simple conclusions and to compare distributions; Understand and identify outliers; Understand and determine skewness. (2.1)(2.4)

Teaching time
8 hours

OBJECTIVES

By the end of the sub-unit, students should:

- know how to interpret diagrams for single variable data;
- be able to identify outliers in data sets and represent these on a box plot;
- Be able to use diagrams to compare distributions;
- Be able to determine skewness.

TEACHING POINTS

Students should be familiar with and be able to interpret histograms, box and whisker plots, stem and leaf diagrams and cumulative frequency diagrams. These should have been covered at International GCSE/GCSE but it is worth a recap for consistency of methods. Also cover calculating summary statistics from diagrams, including the mean and standard deviation from a histogram.

Outliers will need to be identified from data sets and box plots. Any rules to be used will be given in the question, for example $Q_1 - 1.5 \times IQR$, $Q_3 + 1.5 \times IQR$.

Students will need to determine the skewness of a distribution. This can be introduced by looking at the shape of histograms and box plots. Skewness can be determined by comparing the mean, median and mode or by comparing $Q_3 - Q_1$ and $Q_2 - Q_1$.

OPPORTUNITIES FOR PROBLEM SOLVING/MODELLING

Again all of the diagrams and techniques used in this unit could be modelled using real-life data. Students should be encouraged to interpret the diagrams and compare distributions.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Knowing how to interpret statistics students have calculated is sometimes found challenging, and often discriminates between students in exam questions. Full and clear reasons for interpretations and decisions need to be given for marks to be awarded.

Many students have difficulties calculating the sizes of bars in histograms, as commented on by one examiner: 'Most were able to state the correct width of the bar but few used frequency densities correctly to find the height, some finding the frequency density but then calculating $\frac{1}{3} \times 2.5$ rather than $2.5 \div \frac{1}{3}$.

Some identified that 1.5 cm^2 represented 10 customers but were then unable to use this correctly to find the height ... some students had an incorrect class width because they did not realise that the lower class boundary was 70 not 69.5.'

UNIT 2: Probability[Return to overview](#)**SPECIFICATION REFERENCES**

- 3.1** Elementary probability
- 3.2** Sample space. Exclusive and complementary events. Conditional probability
- 3.3** Independence of two events
- 3.4** Sum and product laws

PRIOR KNOWLEDGEInternational GCSE/GCSE (9–1) in Mathematics at Higher Tier

- Record, describe and analyse the frequency of outcomes of probability experiments using tables and frequency trees
- Apply ideas of randomness, fairness and equally likely events to calculate expected outcomes of multiple future experiments
- Relate relative expected frequencies to theoretical probability, using appropriate language and the 0–1 probability scale
- Apply the property that the probabilities of an exhaustive set of outcomes sum to one; apply the property that the probabilities of an exhaustive set of mutually exclusive events sum to one
- Enumerate sets and combinations of sets systematically, using tables, grids, Venn diagrams and tree diagrams
- Construct theoretical possibility spaces for single and combined experiments with equally likely outcomes and use these to calculate theoretical probabilities
- Calculate and interpret conditional probabilities through representation using expected frequencies with two-way tables, tree diagrams and Venn diagrams

KEYWORDS

Sample space, exclusive event, complementary event, discrete random variable, continuous random variable, mathematical modelling, independent, mutually exclusive, Venn diagram, tree diagram, set notation, conditional probability.

2a. Mutually exclusive events; Independent events (3.1) (3.2) (3.3)**Teaching time**

4 hours

OBJECTIVES

By the end of the sub-unit, students should:

- understand and be able to use mutually exclusive and independent events when calculating probabilities.

TEACHING POINTS

Tree and Venn diagrams should have been covered at International GCSE/GCSE but will need to be recapped as one way of looking at probabilities.

The focus at this level is on independent and mutually exclusive events in probability calculations. Students should be confident in the definitions of both independent and mutually exclusive events and how to use their properties to solve real-life probability problems.

At this level independence is shown by showing the product of the probabilities of two events gives the probability of both events occurring together.

OPPORTUNITIES FOR PROBLEM SOLVING/MODELLING

Include questions of the type where A and B are independent which use Venn diagrams and informal use of the addition rule, but where both $P(A)$ and $P(A \cap B)$ for example are unknown; the solution relies on a knowledge of independence.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Students may confuse ‘independent’ and ‘mutually exclusive’.

Using a diagram almost always helps students to answer probability questions. When drawing a Venn diagram, students should remember to include a box defining the universal set.

2b. Using set notation for probability; Conditional probability (3.2) (3.4)

Teaching time
6 hours

OBJECTIVES

By the end of the sub-unit, students should:

- understand and be able to use probability formulae using set notation;
- be able to use tree diagrams, Venn diagrams and two-way tables;
- understand and be able to use the conditional probability formula $P(A|B) = \frac{P(A \cap B)}{P(B)}$.

TEACHING POINTS

Begin by recapping the use of tree diagrams and Venn diagrams, focusing on the use of set notation for probabilities. Introduce the use of two-way tables to find probabilities and use worded questions which are solved most efficiently by forming a two-way table.

Students need to be familiar with and be able to use

$$P(A') = 1 - P(A),$$

$$\text{the addition rule: } P(A \cup B) = P(A) + P(B) - P(A \cap B) \text{ and}$$

$$\text{the conditional probability formula } P(A \cap B) = P(A)P(B|A).$$

Use worded questions where students have to form the set notation as well as questions where the information is already given using set notation.

Ensure the teaching of this section is combined with questions to recap the properties of mutually exclusive and independent events. Make sure these are now answered using set notation too.

OPPORTUNITIES FOR PROBLEM SOLVING/MODELLING

There is scope for using real-life scenarios for probabilities. Ensure that questions are posed where it is not obvious which formulae need to be used.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Mistakes tend to involve the use of the conditional probability formula. For example wrongly assuming independence and putting $P(A) \times P(B)$ rather than $P(A \cap B)$ as the numerator or the incorrect probability in the denominator.

UNIT 3: Correlation and regression

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SPECIFICATION REFERENCES

- 4.1 Scatter diagrams. Linear regression
- 4.2 Explanatory (independent) and response (dependent) variables. Applications and interpretations
- 4.3 The product moment correlation coefficient, its use, interpretation and limitations

PRIOR KNOWLEDGE

International GCSE/GCSE (9–1) in Mathematics at Higher Tier

- Use and interpret scatter graphs of bivariate data; recognise correlation and know that it does not indicate causation; draw estimated lines of best fit; make predictions; interpolate and extrapolate apparent trends while knowing the dangers of doing so.

IAL Mathematics – P1 content

- Straight-line graphs (See P1 SoW Unit 3)

KEYWORDS

Scatter diagram, correlation, positive correlation, negative correlation, no correlation, quadrant, product moment correlation coefficient (PMCC), linear regression, interpolation, extrapolation, coded data, response variable, explanatory variable, dependent variable, independent variable, linear relationship, line of best fit, least squares regression line, bivariate distribution.

3a. Scatter diagrams and least squares regression lines (4.1) (4.2)

Teaching time

9 hours

OBJECTIVES

By the end of the sub-unit, students should:

- recognise the explanatory and response variables;
- know how to interpret scatter diagrams for bivariate data;
- know how to use a calculator to find the equation of the regression line;
- be able to calculate regression coefficients for the equation y on x using standard formulae;
- be able to apply and interpret the least squares regression line of y on x ;
- be able to change the variable in a regression line;
- be able to make predictions using the regression line and understand its limitations;

TEACHING POINTS

The least squares regression line is a way of calculating the equation of a line of best fit.

For bivariate data students should understand the terms explanatory and response variables and know where each is placed on the axes of a scatter diagram. This is particularly important as variables other than y and x are often used. For a regression line of y on x the explanatory variable is the x -variable as it influences the y -variable – think **eXplanatory**. When other variables are used, students should be encouraged to write on the question and change the variables to x and y so they don't get confused. The final answers must be given in terms of the original variables given in the question.

Although derivation of the regression line formula is not required, mathematical software can be used to demonstrate the idea that the sum of the squared residuals is minimised. Calculate b first and then use the exact value of b in the calculation of a . Give final answers to 3 significant figures. Students should be reminded that accuracy marks may be lost due to premature rounding.

The 'S' in S_{xx} stands for 'sum' and should not be confused with Σ . S_{xx} is a shorthand way of writing $\Sigma(x - \bar{x})^2$.

Include questions where the raw data is given as well as questions where only the summary data is provided. Ensure students are familiar with the calculator functions for regression.

A question may ask students to use their equation to estimate a y -value for a given x -value. It is important to check if any sort of *scale* is given in the question. For example, a question may say:

'The table shows data on the number of visitors to the UK in a month, v (1000s), and the amount of money they spent, m (£ millions), for each of 8 months.'

and then go on to require a value to be substituted, which students must be careful with. For example,

'Use your answer to part (d) to estimate the amount of money spent when the number of visitors to the UK in a month is 2 500 000'

means they need to substitute in 2500 for v . Any units of measurement should be given in the final answer.

Students need to understand the use of interpolation when using a regression line equation to make predictions within the range of values of the explanatory variable and they need to understand the dangers of extrapolation (predictions outside the range). A question asking for a prediction will often go on to ask about the reliability of the answer using phrases like, 'comment on the reliability of your estimate' or 'give a reason for your answer'. The response should be either 'Reliable as [the value] lies inside the range of data', or 'Unreliable as [the value] lies outside the range of data', with [the value] being put into the context of the question.

A level Further Mathematics: Statistics 1

Coding is tackled by substituting the coded values into the expression for $y = a + bx$, and rearranging to find an equation in terms of the new variables.

Students may be asked to interpret the gradient, b , and/or the y -intercept, a . This type of question is often answered poorly and it is worth spending extra time on these concepts. For the gradient, students need to say that ‘As [the x ’s] increase by [1 unit], the [y’]s increase/decrease by [b]’, with the bracketed terms put in the context of the question. For the y -intercept, students need to say that ‘The [y] is [a] when [the x] is zero’, again putting the bracketed terms into the context of the question.

OPPORTUNITIES FOR PROBLEM SOLVING/MODELLING

Again all of the diagrams and techniques used in this unit could be modelled using real-life data and related to real-world problems.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Students must be reminded to carefully write down the final equation (not just the values of a and b). ‘Some students accidentally reversed a and b and hence [lost] 5 out of 6 marks’.

Calculations of a and b should be given to 3 significant figures, unless otherwise stated and marks will be lost, for example, for giving an answer of 0.74 rather than 0.740 as required.

It is fairly common to mix up x and y , and so lose marks in an examination. If students work in terms of x and y , the final answers must be given in terms of the original variables given in the question.

Interpretation of the gradient or y -intercept must be contextualised: ‘Invariably students explanations are too vague. General answers such as extrapolation are not good enough to gain marks.’

Questions involving interpretation of the gradient or y -intercept need lots of practice: ‘Many confused the gradient with correlation and simply said that as the number of visitors increased so did the amount of money spent. Those who did get the idea of rate often failed to give the correct numerical values or they had visitors and money the wrong way around, for example “for every £1000 spent the number of visitors increased by 740”.’

The scale given in the question must be taken into account when using the regression line to calculate values. Not doing this can lead to unrealistic answers, for example as one examiner commented: ‘A number of students seemed perfectly content with a final answer of around 30 000 °C here and lost the final mark. Students should be encouraged to try and engage with the context of the questions and this can help them both in interpreting their statistical calculations and assessing the reasonableness of their answers.’

NOTES

Students are not expected to derive formulae for regression coefficients or residuals.

3b. The product moment correlation coefficient (4.3)

Teaching time

7 hours

OBJECTIVES

By the end of the sub-unit, students should:

- be able to calculate the product moment correlation coefficient (PMCC) using their calculators;
- be able to calculate the product moment correlation coefficient (PMCC) using the formula, given summary statistics;
- be able to interpret a correlation coefficient;
- know the effect of coding on the correlation coefficient;

TEACHING POINTS

Students should have an informal appreciation of correlation from International GCSE/GCSE and in this sub-unit they will learn how to calculate and interpret it. They will know the difference between negative and positive correlation and how to interpret it, e.g. ‘as the temperature increases, the sale of ice creams increases’. Emphasise the difference between stating the type of correlation (positive, negative, none) and interpreting it in context.

To help with their informal understanding, a good activity is to get students to guess the values of r from scatter diagrams without doing any calculations.

Students should be taught how to use their calculators to find the PMCC and also to use the standard formulae given in the formula booklet.

Generally, the correlation is considered to be ‘strong’ if $r > 0.7$, or $r < -0.7$. Students should state the strength of correlation as well as whether it is positive or negative.

If asked to draw a scatter diagram, the scales only have to cover the range of values and do not have to start at zero. Pick scales that go up by 1, 2, 5 or 10, or by any of these two multiplied together, e.g. 20.

In order to explain that any linear coding (adding, subtracting, multiplying or dividing) has no effect on r , explain that several people could represent the same data on different scatter diagrams using different scales, and yet the value of r would be the same for each of them. Using linear coding is just like changing the scale. Mathematical software can also be used to demonstrate this.

Coding does, however, affect the values of S_{xx} , S_{yy} and S_{xy} . These are affected in the same way as variance.

For example, if $t = 5x - 2$, then $S_{tt} = 5^2 \times S_{xx}$

Often variables are not given in terms of x and y , and students should be encouraged to write on the question and change the variables to x and y so they don’t get confused. The final answers must be given in terms of the variables originally given in the question.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Students must learn to engage with each question, and not just give text book answers. Answers should always be put in the context of the question.

A level Further Mathematics: Statistics 1

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Students should give answers for r to 3 significant figures unless otherwise stated and marks will be lost if answers are not given to the correct level of accuracy.

Care must be taken when looking at the context of the question. Finding a reasonably high correlation does not automatically mean that it is relevant: ‘Most students realised there was positive correlation, but some went on to state that this suggested support for the researcher’s belief and only the more astute explained that the researcher should have been expecting a negative correlation and these data therefore did not offer support’.

Students must know when their value of r will support the use of a regression equation: ‘Many stated that there was negative correlation (although some thought this meant that the use of a regression equation was *not* suitable) but few stated clearly that the use of a regression equation was suitable because there was *strong* correlation. Some simply said that “the points were close to a straight line” but there was no scatter diagram to support this and without a clear statement that the strong correlation suggests this the examiners could not award the mark’.

‘Most gave a comment about strong correlation or commented that the value of r was close to 1 but some misinterpreted the question and explained why linear regression was a useful tool rather than justifying its use in this situation’.

NOTES

Although students can find the value of the correlation coefficient by entering data into a calculator, the raw data will not always be provided and they will be expected to be able to apply the formula given summary statistics.

UNIT 4: Discrete random variables

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SPECIFICATION REFERENCES

- 5.1 The concept of a discrete random variable
- 5.2 The probability function and the cumulative distribution function for a discrete random variable
- 5.3 Mean and variance of a discrete random variable
- 5.4 The discrete uniform distribution

PRIOR KNOWLEDGE

- An understanding of probability from the earlier unit (See Unit 2)

International GCSE/GCSE (9–1) in Mathematics at Higher Tier

- Order positive and negative integers, decimals and fractions; use the symbols $=$, $\neq$, $<$, $>$, $\leq$, and $\geq$

KEYWORDS

Probability, discrete distribution, discrete random variable, uniform, cumulative probabilities, variable, sample space, probability distribution, probability function, mean, expectation, expected value, standard deviation, variance.

4a. Use a discrete probability distributions to model simple situations; Identify the discrete uniform distribution (5.1) (5.2) (5.4)

Teaching time
6 hours

OBJECTIVES

By the end of the sub-unit, students should:

- understand and be able to use simple, discrete probability distributions;
- understand and be able to use the cumulative distribution function for a discrete random variable;
- be able to identify the discrete uniform distribution.

TEACHING POINTS

A random variable is discrete if a list can be made of all the possible values the variable could take. A probability distribution is usually given in the form of a table showing the probability $P(X = x)$ of each value occurring. The corresponding probability function is given as a function $p(x)$, piecewise, for discrete values of x , and can be turned into a probability distribution table.

Remember $p(x)$ is the same as $P(X = x)$.

Students should know that $\sum p(x) = 1$ and they will be expected to model simple situations such as the score on a biased dice by using discrete probability distributions. Look at equally likely outcomes such as the score on a fair dice and the associated probability function for the discrete uniform distribution.

Use the cumulative distribution function $F(x_0) = P(X \leq x_0)$ and make sure that students are familiar with the notation $F(x_0)$. Look at problems where students are asked to find the probability function from the cumulative distribution function.

OPPORTUNITIES FOR PROBLEM SOLVING/MODELLING

Look at a variety of real-world scenarios and word problems, and model using a discrete random variable.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Many students do not recognise the cumulative distribution function $F(x)$, confusing, for example, $F(5)$ with $P(X = 5)$, or assuming that the sum of the $F(x)$ values equal to 1.

When errors in calculations lead to negative values for the probabilities some candidates still use these impossible values in later stages of their working, not realising that an error has been made and then trying to correct it.

NOTES

Make sure students are familiar with the notation $F(x_0)$.

4b. Mean and variance of discrete probability distributions (5.3)

Teaching time

7 hours

OBJECTIVES

By the end of the sub-unit, students should:

- be able to calculate the mean and variance of discrete probability distributions using $E(X) = \mu = \sum xP(X = x)$ and $\text{Var}(X) = \sigma^2 = \sum x^2P(X = x) - \mu^2$;
- know how to find the expectation of a linear function of a random variable.

TEACHING POINTS

The formula, $E(X) = \sum xp(x)$, for the expected value can be derived from the formula for the mean, $\bar{x} = \frac{\sum fx}{\sum f}$. The formula for $\text{Var}(X)$ can also be written as $\text{Var}(X) = E(X^2) - (E(X))^2$ and students may be asked to calculate $E(X^2)$ when given $\text{Var}(X)$ and $E(X)$.

Students should know that:

- $E(aX) = aE(X)$
- $E(X + b) = E(X) + b$
- $E(aX + b) = aE(X) + b$
- $\text{Var}(aX) = a^2\text{Var}(X)$
- $\text{Var}(X + b) = \text{Var}(X)$
- $\text{Var}(aX + b) = a^2\text{Var}(X)$.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Questions may require students to use the calculations for $E(X)$ and $\text{Var}(X)$ to assess the suitability of models.

Students may be asked to work out if a game is fair, and to calculate expected winnings.

Students could also be encouraged to prove algebraically that

$$E(aX + b) = aE(X) + b \text{ and } \text{Var}(aX + b) = a^2\text{Var}(X).$$

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Students need to have a clear understanding of the formula for finding $\text{Var}(X)$; mistakes include forgetting to square the mean before subtracting or thinking that $\text{Var}(X) = E(X^2)$.

Students should be careful when evaluating $\text{Var}(aX)$, particularly when a is negative and not make the mistake, for example, of finding $-15^2 \times 0.064$ rather than $(-15)^2 \times 0.064$.

UNIT 5: The Normal distribution
Understand and use the Normal distribution (1.1) (6.1)**Teaching time**
8 hours[Return to overview](#)**SPECIFICATION REFERENCES**

- 6.1** The Normal distribution including the mean, variance and use of tables of the cumulative distribution function.

PRIOR KNOWLEDGE

- Probability calculations, independent events, conditional probability (See Unit 2)
- International GCSE/GCSE (9–1) in Mathematics at Higher Tier
- Solve two simultaneous equations in two variables (linear/linear or linear/quadratic) algebraically

KEYWORDS

Continuous distribution, cumulative probabilities Normal distribution, standard normal, mean, variance, histogram, probability distribution.

OBJECTIVES

By the end of the unit, students should:

- understand the properties of the Normal distribution;
- be able to find probabilities using the Normal distribution.

TEACHING POINTS

The Normal distribution needs to be linked to histograms and the mean. A good way to introduce the topic is to look at heights on a histogram and show how it can be smoothed into the Normal distribution curve, stating this is due to the Normal being a continuous distribution.

Discuss all the properties of the Normal distribution, making sure students are confident with the symmetry of the distribution, that mean = mode = median and the asymptotic nature of the bell-shaped curve. Cover the proportions of data within 1, 2 and 3 standard deviations of the mean and discuss the idea that the area under the curve is 1.

Students should understand the notation $X \sim N(\mu, \sigma^2)$ for the Normal distribution.

Students should be shown how to find the probabilities from Normal distributions using their calculators.

Students do need to know the standardisation formula $Z = \frac{x - \mu}{\sigma}$ and be able to transform X values to Z values. Be clear that the denominator is the standard deviation rather than the variance which may be given. Students should be encouraged to draw diagrams to represent the distribution and use this to check (at least for $>$ or < 0.5) the probability they find.

Diagrams will also help students when working backwards from a probability to find a Z value, a diagram will indicate whether the Z value is positive or negative. Again, students should be shown how to use their calculator to find these values.

Questions may involve the use of linear simultaneous equations to find for example both the mean and standard deviation of the Normal distribution.

A level Further Mathematics: Statistics 1

You should recap the probability of independent events as this can be incorporated into questions involving the Normal distribution.

OPPORTUNITIES FOR PROBLEM SOLVING/MODELLING

Use plenty of reverse problem examples worded in a variety of ways. Ensure students can find Z values for quartiles and percentiles.

This would be a good opportunity to link the work done so far in this module and discuss mathematical modelling in statistics and probability. Discuss the process of setting up a model, collecting data, using the model to make predictions, making comparisons against the model and refining the model. Examples can be drawn from real-world problems and discuss how they can be modelled by a Normal distribution, a discrete uniform distribution or a regression line.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Main errors are due to confusion between probabilities and Z values, particularly when it comes to notation, and not using the full four decimal place accuracy in calculations.

An emphasis on using diagrams alongside the calculations should help address some of the difficulties.

NOTES

Knowledge of the probability density function is not required, neither are any derivations of mean, variance or the cumulative distribution function.