

INTERNATIONAL ADVANCED LEVEL

MATHEMATICS/ FURTHER MATHEMATICS/ PURE MATHEMATICS

SCHEME OF WORK

Mechanics 1

Pearson Edexcel International Advanced Subsidiary in Mathematics (XMA01)

Pearson Edexcel International Advanced Subsidiary in Further Mathematics (XFM01)

Pearson Edexcel International Advanced Subsidiary in Pure Mathematics (XPM01)

Pearson Edexcel International Advanced Level in Mathematics (YMA01)

Pearson Edexcel International Advanced Level in Further Mathematics (YFM01) Pearson

Edexcel International Advanced Level in Pure Mathematics (YPM01)

First teaching September 2018

First examination from January 2019

First certification from August 2019 (International Advanced Subsidiary) and August 2020 (International Advanced Level)



A level Mathematics: Mechanics 1

Mechanics 1

Unit	Title	Estimated hours
1	Quantities and units in mechanics: Introduction to mathematical modelling and standard S.I. units of length, time and mass	1
2	Vectors in mechanics	
<u>a</u>	Definitions, magnitude/direction, addition and scalar multiplication	7
<u>b</u>	Position vectors, distance between two points, application of vectors to displacement, velocity, acceleration and forces	7
3	Kinematics of a particle moving in a straight line	
<u>a</u>	Graphical representation of velocity, acceleration and displacement	5
<u>b</u>	Motion in a straight line under constant acceleration; <i>suvat</i> formulae for constant acceleration; Vertical motion under gravity	6
4	Forces and Newton's laws	
<u>a</u>	Newton's first law, Newton's third law, force diagrams	3
<u>b</u>	Newton's second law, ' $F = ma$ ', resolving forces, connected particles, problems involving smooth pulleys	8
<u>c</u>	Momentum and impulse; derivation of units and formulae Impulse-momentum principle. Conservation of momentum applied to collisions and 'jerking' string problems	8
<u>d</u>	Friction forces (including coefficient of friction μ)	4
5	Statics of a particle: Equilibrium, Forces in vector form, Maximum value of the frictional force	4
6	Moments: Forces' turning effects	7
		60 hours

A level Mathematics: Mechanics 1

UNIT 1: Quantities and units in mechanics

Introduction to mathematical modelling and standard S.I units of length, time and mass (1.1)

Teaching time

1 hour

[Return to overview](#)

SPECIFICATION REFERENCES

1.1 The basic ideas of mathematical modelling as applied in Mechanics

PRIOR KNOWLEDGE

IGCSE/GCSE (9-1) in Mathematics at Higher Tier

- Change freely between related standard units (e.g. time, length, area, volume/capacity, mass) and compound units (e.g. speed, rates of pay, prices, density, pressure) in numerical and algebraic contexts
- Use compound units such as speed, rates of pay, unit pricing, density and pressure
- Plot and interpret graphs (including reciprocal graphs and exponential graphs) and graphs of non-standard functions in real contexts to find approximate solutions to problems such as simple kinematic problems involving distance, speed and acceleration
- Calculate or estimate gradients of graphs and area under graphs (including quadratic and non-linear graphs), and interpret results in cases such as distance-time graphs, velocity-time graphs and graphs in financial contexts

KEYWORDS

Modelling, smooth, rough, light, inelastic, inextensible, uniform, particle, rigid body, mass, weight, rod, plane, peg, pulley, bead, wire, lamina, length, distance (m), displacement (m), velocity (m s^{-1}), speed (m s^{-1}), acceleration (m s^{-2}), force (N), retardation (m s^{-2}), newtons (N), scalar, vector, direction, magnitude, (normal) reaction, friction, tension, thrust, compression.

NOTES

There may not be a direct examination question on this topic. However, the modelling process and fluent knowledge of the S.I. units is a vital pre-requisite that underpins the rest of the mechanics course.

OBJECTIVES

By the end of the sub-unit, students should:

- understand the concept of a mathematical model, and be able to abstract from a real-world situation to a mathematical description (model);
- know the language used to describe simplifying assumptions;
- understand the particle model;
- be familiar with the basic terminology for mechanics;
- be familiar with commonly-made assumptions when using these models;
- be able to analyse the model appropriately, and interpret and communicate the implications of the analysis in terms of the situation being modelled;
- understand and use fundamental quantities and units in the S.I. system: length, time and mass;
- understand that units behave in the same way as algebraic quantities, e.g. meters per second is $\text{m/s} = \text{m} \times \frac{1}{\text{s}} = \text{m s}^{-1}$.

A level Mathematics: Mechanics 1

TEACHING POINTS

Begin by asking students ‘What is mechanics?’ Lead them to the idea that mechanics is a branch of applied mathematics that deals with motion and the forces producing motion.

Students need to be comfortable with the idea that mathematics is used to model real life and need to become familiar with the modelling cycle:

mechanics problem → create a mathematical model (using diagrams, general principles or formulae) → solve the model → refer back to the original problem → refine the model

[You could link with the data-handling cycle.]

It is important for students to get a ‘feel’ for mechanics at this early stage in order to support later work.

OPPORTUNITIES FOR PROBLEM SOLVING/MODELLING

Examples of problems that may be solved in this way include:

- How far apart should the cameras be within an average speed zone?
- At what angle should you hold an umbrella to keep snow off you?

Some examples of simplifying assumptions for these problems include:

- treating the car as a particle
- motion is in a straight line
- snow falls vertically.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Students can generally correctly state assumptions, but they need to make sure that any assumptions or statements about the model relate directly to the context they are considering. For example they could make the comment ‘the resistance will not be constant’ more specific by saying ‘resistance will increase as velocity increases’.

NOTES

The particle (point mass) model is introduced here, i.e. the body has no size but does have mass, so rotation is ignored and the forces all act at one point.

The language of simplifying assumptions (light, smooth, uniform, inextensible, thin, rigid etc.) is mostly introduced in subsequent sections.

A level Mathematics: Mechanics 1

UNIT 2: Vectors in mechanics

[Return to overview](#)

SPECIFICATION REFERENCES

- 2.1** Magnitude and direction of a vector. Resultant of vectors may also be required
- 2.2** Application of vectors to displacements, velocities, accelerations and forces in a plane

PRIOR KNOWLEDGE

IAS Mathematics – P1 content

- Surds (Unit 1 of the P1 SoW)

IGCSE/GCSE (9-1) in Mathematics at Higher Tier

- Vectors
- Use bearings
- Know the formula for Pythagoras' theorem and apply the trigonometrical ratios to find angles and lengths

KEYWORDS

Vector, scalar, magnitude, direction, component, parallel, perpendicular, modulus, dimension, ratio, collinear, scalar product, position vectors.

A level Mathematics: Mechanics 1

2a. Definitions, magnitude/direction, addition and scalar multiplication (2.1)

Teaching time
7 hours

OBJECTIVES

By the end of the sub-unit, students should:

- be able to use vectors in two dimensions;
- be able to calculate the magnitude and direction of a vector and convert between component form and magnitude/direction form;
- be able to add vectors diagrammatically and perform the algebraic operations of vector addition and multiplication by scalars, and understand their geometrical interpretations.

TEACHING POINTS

Students need to be familiar with column vectors and with the use of **i** and **j** vectors in two dimensions.

Students should be able to find a unit vector in the direction of **a**, and be familiar with the notation $|\mathbf{a}|$.

The triangle and parallelogram laws of addition should be known and students should be able to use them.

Students should understand that vectors are commutative.

Where answers are given in surds they should be simplified if possible, or rounded to 3 s.f.

When performing operations on vectors this should also be understood geometrically, diagrams will be helpful here. Students should be able to use given diagrams but also draw their own in order to assist with questions.

Students should understand and be able to use the conditions for parallel vectors.

Use the classroom floor as a 2-dimensional grid to help students visualise vectors. Use the position of students in the room to illustrate concepts.

Consider vectors in mechanics, for example forces, velocities, displacements.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Students can prove vectors are parallel to demonstrate their reasoning skill.

Given particular vectors, students can investigate places they can or cannot reach, for example the knights problem on a chessboard.

Consider an aircraft landing in a cross-wind – what direction does it need to fly?

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Students sometimes make mistakes when manipulating vectors in **i** and **j** form and should be encouraged to use column vectors when possible.

A level Mathematics: Mechanics 1

2b. Position vectors, distance between two points, application of vectors to displacement, velocity, acceleration and forces (2.2)

Teaching time
7 hours

OBJECTIVES

By the end of the sub-unit, students should:

- understand and be able to use position vectors;
- be able to calculate the distance between two points represented by position vectors;
- be able to use vectors to solve problems in mechanics.

TEACHING POINTS

Students should know and be able to use $\overrightarrow{OB} - \overrightarrow{OA} = \overrightarrow{AB} = \mathbf{b} - \mathbf{a}$

Students should be able to calculate the distance between two points (x_1, y_1) and (x_2, y_2) using the formula $d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$.

They should know that force, velocity and constant acceleration are vectors, and that speed is the magnitude of the velocity vector.

Begin by walking across the room and explaining the difference between position (referred to a fixed origin), displacement (a vector measured from any position) and distance (a scalar quantity for the total movement). Then move onto discussing speed (the rate at which an object covers distance) and velocity (the rate of change of displacement or speed in a certain direction).

Look at problems involving displacement, velocity, acceleration and time, such as 'A particle is moving with constant velocity $\mathbf{v} = 2\mathbf{i} + 3\mathbf{j} \text{ ms}^{-1}$. If its initial position is $\mathbf{r} = \mathbf{i} - 2\mathbf{j}$ find its position after 5 seconds.'

Point out to students that the position of a particle at time t seconds is given by $\mathbf{r} = \mathbf{r}_0 + t\mathbf{v}$, where $\mathbf{r}_0$ is the initial position and $\mathbf{v}$ is the velocity. Revise bearings and provide problems in which a position is given or required as a bearing.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Solve problems in context, such as determining whether two ships moving in different directions will collide, or finding the minimum distance between the two ships.

Find the position vector of the fourth corner of a shape (e.g. parallelogram) $ABCD$ with three given position vectors for the corners A , B and C .

Use regular polygons to find vectors connecting different vertices.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Examiners comment that students understand the simple basics of vectors but sign errors are fairly common at some stage of the working. Other comments include 'a number of candidates fail to find the magnitude of their position vector or velocity vector to obtain the distance or speed as required' and 'many candidates do not appreciate the difference between velocity and speed and some struggle to convert angles to bearings'.

A level Mathematics: Mechanics 1

UNIT 3: Kinematics of a particle moving in a straight line

[Return to overview](#)

SPECIFICATION REFERENCES

3.1 Motion in a straight line with constant acceleration

PRIOR KNOWLEDGE

IGCSE/GCSE (9-1) in Mathematics at Higher Tier

- Change freely between related standard units (e.g. time, length, area, volume/capacity, mass) and compound units (e.g. speed, rates of pay, prices, density, pressure) in numerical and algebraic contexts
- Use compound units such as speed, rates of pay, unit pricing, density and pressure
- Substitute numerical values into formulae and expressions, including scientific formulae
- Understand and use standard mathematical formulae; rearrange formulae to change the subject
- Plot and interpret graphs (including reciprocal graphs and exponential graphs) and graphs of non-standard functions in real contexts to find approximate solutions to problems such as simple kinematic problems involving distance, speed and acceleration
- Calculate or estimate gradients of graphs and area under graphs (including quadratic and non-linear graphs), and interpret results in cases such as distance-time graphs, velocity-time graphs and graphs in financial contexts
- Solve linear equations in one unknown algebraically (including those with the unknown on both sides of the equation)
- Solve quadratic equations (including those that require rearrangement) algebraically by factorising, by completing the square and by using the quadratic formula

IAS Mathematics – Pure Mathematics 1 content

- Gradient (Unit 3 of the P1 SoW)

KEYWORDS

Distance (m), displacement (m), speed (m s^{-1}), velocity (m s^{-1}), acceleration (m s^{-2}), retardation (m s^{-2}), deceleration (m s^{-2}), scalar, vector, 2D, linear, area, trapezium, gradient, equations of motion, gravity, constant, 9.8 m s^{-2} , vertical.

NOTES

The guidance on the specification document states that graphical solutions to problems may be required. This section assumes *constant* acceleration; hence the graphical approach involves linear line segments and the familiar equations of linear motion (*suvat* formulae) for constant acceleration. (N.B. ‘the equation of motion’ refers to $F = ma$, and is nothing to do with these formulae).

The usual value for g in this course is 9.8 m s^{-2} , but some questions may specify a different value, or that a final answer should be given in terms of g . Students may assume that g is constant, but should be aware that it is not a universal constant but depends on location.

A level Mathematics: Mechanics 1

3a. Graphical representation of velocity, acceleration and displacement (3.1)

Teaching time
5 hours

OBJECTIVES

By the end of the sub-unit, students should:

- be able to draw and interpret kinematics graphs, knowing the significance (where appropriate) of their gradients and the areas underneath them.

TEACHING POINTS

Introduce this topic by making links to the IGCSE/GCSE (9-1) in Mathematics prior knowledge for distance–time (travel) graphs and speed–time graphs. Kinematics is the analysis of a particle’s motion without reference to the resultant force that caused that motion.

Stress that forces causing the motion of the body in this section are *constant*, therefore acceleration is constant and this results in a *straight line* speed–time or velocity–time graph.

Extend the ideas to displacement by considering a particle which moves in reverse direction back beyond the starting point.

For a velocity–time graph, consider the units for the area of a unit square 1 m s^{-1} by 1 s . The ‘s’ cancels, leaving ‘m’, therefore the area represents the displacement.

Discuss and interpret graphs that model real situations. For example, the distance–time graph for a particle moving with constant speed, the velocity–time graph for a particle with constant acceleration.

OPPORTUNITIES FOR PROBLEM SOLVING/MODELLING

Throw an object straight (vertically) up in the air. Time the flight and estimate the greatest height, to scale the graphs correctly, and keep for possible later use. Draw the displacement–time and velocity–time graphs (upward direction positive and initial velocity non-zero). If the object is caught at the same height at which it was thrown, what is the average velocity for the motion?

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Many students can draw a velocity–time graph with the correct shape, but do not always label the required speeds and times clearly on the axes. Students often tend to add a scale (for example 4, 8, 12, 16, ...) unnecessarily, rather than just indicating the initial and final speeds.

Candidates are able to find distance travelled and the acceleration from velocity–time graphs and can find an average speed, but some struggle with the vocabulary of velocity and displacement.

NOTES

This unit can be linked with Unit 3b by drawing a general velocity–time graph for a particle with initial velocity (u), final velocity (v), taking time (t) moving under constant acceleration (a). The gradient of the line is $\frac{v-u}{t} = a$, which rearranges to $v = u + at$. Finding the area under the graph in three different ways will lead to three of the other *suvat* formulae, but $v^2 = u^2 + 2as$ will have to be derived by eliminating t between two of them.

A level Mathematics: Mechanics 1

3b. Motion in a straight line under constant acceleration; *suvat* formulae for constant acceleration; Vertical motion under gravity (3.1)

Teaching time
6 hours

OBJECTIVES

By the end of the sub-unit, students should:

- recognise when it is appropriate to use the *suvat* formulae for constant acceleration;
- be able to solve kinematics problems using constant acceleration formulae;
- be able to solve problems involving vertical motion under gravity.

TEACHING POINTS

Make links back to Unit 3a and contrast the previous graphical approach with this algebraic approach. Note that there are five quantities, s , u , v , a and t (four vectors and one scalar) and each formula relates four of them hence there are five formulae. The formulae that must be derived and learnt are:

- $v = u + at$
- $s = \frac{(u+v)t}{2}$
- $s = ut + \frac{1}{2}at^2$
- $v^2 = u^2 + 2as$
- $s = vt - \frac{1}{2}at^2$

These formulae are only valid for *constant* acceleration in a straight line (and are referred to as the *suvat* formulae).

When solving problems, write down known variables and the variable(s) to be found – this should help to identify which one (or more, as some problems will involve simultaneous equations) of the *suvat* formulae to select. Emphasise to students the need to make sure units are compatible.

Model the good practice of drawing a diagram to illustrate the situation whenever possible, especially when considering vertical motion under gravity. This will encourage students to draw their own diagrams.

Mark the positive direction on the diagram and take acceleration due to gravity (g) to be 9.8 m s^{-2} unless directed otherwise. Students may assume that g is constant, but they should be aware that g is not a universal constant but depends on location.

If an object is thrown upwards and upwards is taken as being positive then $a = -9.8 \text{ m s}^{-2}$. Explain that the velocity is zero at the greatest height and there is symmetry in the path (up and down to the same point) due to the fact that we model air resistance as being negligible.

OPPORTUNITIES FOR PROBLEM SOLVING/MODELLING

One of the more demanding problems is when two objects are released (or dropped) at different times, say 2 seconds apart, and students are asked to find the common position when one catches-up or passes the other. Students may find it difficult to select the times (values of t) to assign in the equations; they may need guiding towards t and $(t - 2)$ or t and $(t + 2)$.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Students are generally able to use *suvat* formulae in 2D to find unknown heights, velocities etc. However, students sometimes ignore the significance of a negative value for velocity, acceleration or displacement and don't refer their answer back to the original problem. They need to recognise that $s = -3 \text{ m}$ means the object is 3 m *below* its starting point in the negative direction i.e. s is effectively a coordinate. This is where a diagram helps students understand the physics of the situation.

A level Mathematics: Mechanics 1

UNIT 4: Forces and Newton's laws

[Return to overview](#)

SPECIFICATION REFERENCES

- 4.1 The concept of a force. Newton's laws of motion
- 4.2 Simple applications including the motion of two connected particles
- 4.3 Momentum and impulse. The impulse-momentum principle. The principle of conservation of momentum applied to two particles colliding directly
- 4.4 Coefficient of friction

PRIOR KNOWLEDGE

- Modelling and definitions/assumptions from the introduction in Unit 1
- Vectors in mechanics (See Unit 2)

IGCSE/GCSE (9-1) in Mathematics at Higher Tier

- Solve two simultaneous equations in two variables (linear/linear or linear/quadratic) algebraically; find approximate solutions using a graph

KEYWORDS

Force, newtons, mass, weight, gravity, tension, thrust, compression, air resistance, reaction, driving force, braking force, resultant, force diagram, equilibrium, inextensible, light, negligible, particle, smooth, uniform, pulley, string, retardation, free particle, velocity, speed, Ns, momentum, collisions, direct, smooth, body, sphere, coalesce, conservation, friction, coefficient of friction, μ , limiting.

NOTES

This section introduces the idea of resolving forces in two perpendicular directions; a skill that will be required throughout the mechanics modules.

A level Mathematics: Mechanics 1

4a. Newton's first law, Newton's third law, force diagrams (4.1)

Teaching time

3 hours

OBJECTIVES

By the end of the sub-unit, students should:

- understand the concept of a force;
- understand the language relating to forces;
- be able to identify the forces acting on a particle and represent them in a force diagram;
- understand and use Newton's first law;
- understand and use Newton's third law.

TEACHING POINTS

Review the vector quantities displacement and velocity as the vector versions of distance and speed respectively.

Mention the special acceleration (for a falling object) due to gravity. In this course, this value is assumed to be a constant g , usually 9.8 m s^{-2} though it does vary in the real world.

This could be a good opportunity to dispel common misconceptions around weight and mass. Make it clear that mass is the amount of 'stuff' something is made of, is a scalar and is fixed (in kg), whereas weight is a force of attraction between an object and the centre of the earth and can vary depending on gravity and is measured in newtons. Hence weight = mass $\times$ gravity (or $W = mg$).

Newton said '*An object continues in state of rest or uniform motion unless acted on by an external force.*' Hence one can define a force as something which causes a body to accelerate. Explain to students that 'no force acting' means a body will either be stationary or be moving with constant velocity (i.e. acceleration = zero). This is why in outer space an object keeps moving at constant speed once pushed (there are no forces to speed it up, slow it down or stop it moving).

So, an object at rest or constant velocity $\Rightarrow$ no resultant force; an object changing speed or direction $\Rightarrow$ resultant force. This will lead to Newton's second law in the next section.

Newton also stated '*When an object A exerts a force on another object B there is an equal and opposite reaction force of B on A.*' This is Newton's third law. Explain that if a book is on a smooth, horizontal table, the forces acting on the book are the weight, W (vertically down) and the normal reaction, R (always at 90° to the *surface* of contact). Assuming the table surface material is strong enough to hold the full weight of the book, the two forces balance each other and there is no resultant force. The book does not move, hence it is in equilibrium.

Ask questions such as: If the book has a mass of 5 kg, what is its weight? Therefore, what would the magnitude of the normal reaction be to guarantee equilibrium?

Draw different examples of force diagrams to illustrate: weight, reaction, tension (in strings), thrust (in rods), compression (in light rods, springs) etc.

To illustrate thrust, balance a book on a ruler. In which direction is the thrust force acting?

Begin with examples where forces are acting in one dimension, then move onto examples where forces are acting in perpendicular directions. Encourage students to draw diagrams, to identify all the forces acting on the object and to mark these clearly on their diagram.

A level Mathematics: Mechanics 1

OPPORTUNITIES FOR PROBLEM SOLVING/MODELLING

Practise modelling real-world situations using the ideas from Unit 1, and discuss the assumptions that have been made in the context of the problem. Students could consider whether the assumptions are reasonable and they should draw force diagrams. Problems could be introduced where the forces are not in two perpendicular directions.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Students are often good at drawing force diagrams, but common errors are omitting arrowheads, incorrectly labelling (e.g. 4 kg rather than 4g) and missing off the normal reaction.

Students can easily be confused by the vocabulary, e.g. mixing up 'resultant' and 'reaction' or 'mass' and 'weight'. It is important to be really clear when giving the definitions and to always use the correct vocabulary in discussions.

A level Mathematics: Mechanics 1

4b. Newton's second law, ' $F = ma$ ', resolving forces, connected particles, problems involving smooth pulleys (4.1) (4.2)

Teaching time
8 hours

OBJECTIVES

By the end of the sub-unit, students should:

- understand and be able to use Newton's second law for motion in a straight line (1D and 2D problems);
- be able to resolve a force into components and be able to select suitable directions for resolution;
- be able to apply Newton's second law to problems involving smooth pulleys and connected particles.

TEACHING POINTS

Newton stated, '*Where there is a force, there is an acceleration (or deviation from uniform motion) and the force is proportional to the acceleration*'. Therefore $F \propto a$, and choosing the constant to suit the motion units gives $F = ma$ (Newton's second law). This is known as the 'equation of motion'.

Explain to students that if they sum all the effects of the forces acting, in a particular direction, this will be equal to the mass $\times$ the acceleration in that direction. This process is called resolving the forces in that direction e.g. resolving horizontally, or $R(\rightarrow)$ for short. It's usually best to resolve in the direction of the acceleration and/or perpendicular to the direction of the acceleration.

When resolving always take the positive direction as the direction of the acceleration and put all the forces on one side of the equation and (mass $\times$ acceleration) on the other side.

Begin by considering situations where forces are acting at right angles to one another (horizontal and vertical) and resolve the system using $R(\rightarrow)$ and $R(\uparrow)$ to create two equations. Extend to situations where the forces are not perpendicular to each other, and guide students to consider the right-angled triangle and use trigonometry to show that the horizontal component of a force F is ' $F \cos \theta$ ' and the vertical component is ' $F \sin \theta$ '. Move onto inclined planes and show that the weight component of a particle acts in two directions: along and perpendicular to the plane.

When working on connected particles problems (such as trains or pulley systems) explain to students that they should consider the whole system as well as the separate parts. Applications to be covered are lift problems, car and caravan type questions and connected particles passing over a smooth pulley. Consider both pulley scenarios: a pulley with both strings hanging vertically; and a pulley at the end of a horizontal table.

OPPORTUNITIES FOR PROBLEM SOLVING/MODELLING

For the connected particle problems, discuss, the assumptions from Unit 1, i.e. smooth pulley, inextensible string, same tension in the string. Extend the questions so that (for a pulley question) the particle moving down eventually hits the ground and the string goes slack. This means the particle moving up continues as a 'free' particle so we now apply the equations of motion with $a = -9.8 \text{ m s}^{-2}$.

Include more challenging problems, such as smooth inclined planes where the forces are not perpendicular to each other.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Pulleys: In past exam questions, most students used an equation of motion for each particle with very few 'single equation' solutions. Students may also mistakenly take the acceleration to be equal to g rather than the value obtained in the question.

Two vehicles: In exam questions of a car-and-trailer type, students may consider the car and trailer as a single system. Common errors when resolving are: to add a tension force (when there is no rope); to consider the weight; or to confuse the positive and negative directions.

A level Mathematics: Mechanics 1

NOTES

Emphasise the importance of drawing clearly labelled diagrams and of thinking carefully about direction of the acceleration and each of the forces.

A level Mathematics: Mechanics 1

4c. Momentum and impulse; derivation of units and formulae. Momentum-impulse principle. Conservation of momentum applied to collisions and 'jerking' string problems (4.3)

Teaching time
8 hours

OBJECTIVES

By the end of the sub-unit, students should:

- understand the definitions, derivation, and units of momentum and impulse;
- understand what happens to the momentum of a particle as a result of a collision;
- be able to use the principle of conservation of momentum applied to direct collisions and 'jerking' string problems
- in one dimension.

TEACHING POINTS

Begin with two equations, familiar from the previous sections:

$$v = u + at \quad \text{and} \quad F = ma.$$

Rearrange the latter to make $a = \frac{F}{m}$, substitute into $v = u + at$ and then rearrange to give

$$Ft = mv - mu.$$

Note that we are assuming F is constant in this equation.

What is the quantity mass $\times$ velocity? Its unit is based on the Ft part of the formula above which gives newtons $\times$ seconds (N s).

Momentum is defined as mass $\times$ velocity and it is basically a measure of a body's capacity to keep moving (or stop moving).

The fact that the formula has initial and final velocities suggests a change of velocity, which means that an acceleration exists, therefore a force must cause that change. However, this could be a very sudden change, due to a collision or a strike. The quantity Ft is defined as the impulse of the force.

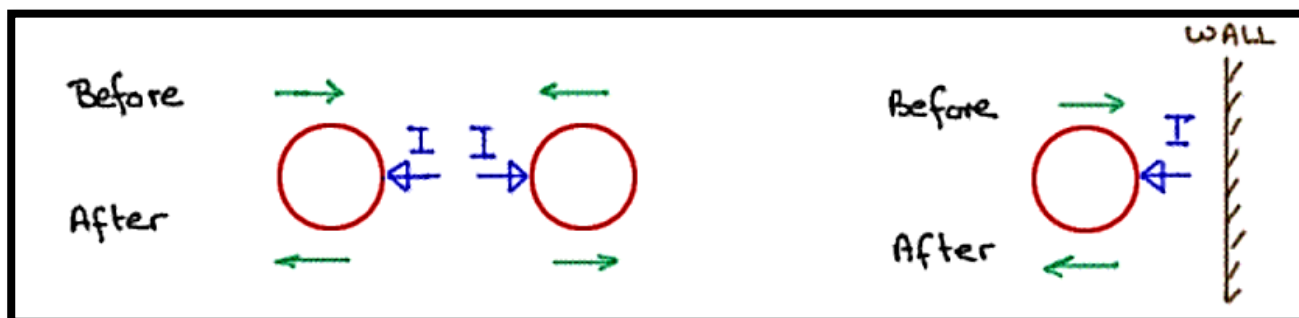
This gives us the impulse-momentum principle which states:

$$\text{Impulse} = \text{change in momentum for a body } (I = mv - mu)$$

Note that velocities are vectors which therefore, in one dimension, could be positive or negative depending on their directions; this means that the corresponding impulse could also be positive or negative.

The idea of an impulse can be illustrated by considering a hammer hitting a nail. Each strike on the nail is for a very short time, so the quantity Ft will have a large value of F (as the t is very small). This is why the hammer is effective. F is called an 'impulsive' force.

Consider a single body reversing direction after being hit (e.g. a cricket ball hit by a bat) or bouncing off a vertical wall. The impulse can be calculated using the impulse-momentum principle (i.e. impulse = change in momentum for the ball).



A level Mathematics: Mechanics 1

The formula for impulse ($Ft = mv - mu$) is a consequence of Newton's second law. During a collision a force acts on each particle for a short time when the particles are in contact. By Newton's third law these forces are of equal magnitude and act in opposite directions. So the impulses acting on the particles are also of equal magnitude and acting in opposite directions.

Hence: $m_1(v_1 - u_1) = -m_2(v_2 - u_2)$

This can be rearranged to give the law of conservation of (linear) momentum for a collision which is of fundamental importance in mechanics, namely: $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

In order to illustrate this you could show some clips of collisions between two vehicles from the internet (e.g. car racing crash) or balls in a game of snooker.

Students could experiment by flicking coins (one large and one small) to see how they move after collisions. (Sometimes the direction of the velocity is reversed if a small coin strikes a much heavier coin.)

This is also a good opportunity to mention the other assumptions we need to make for this topic. We model the colliding bodies as particles; this ensures that collisions are direct and there is no spin. Generally the surface on which they are moving is modelled as smooth and the particles will be travelling along the same straight line. However, remember that the smooth surface only affects the motion before and after the collision and not the impact itself. For collisions of two particles, it is important that students draw diagrams which cover before and after the collision; the velocities, masses and direction of impulse should be included as appropriate for each particle. The law of conservation of momentum will apply to the colliding particles.

Momentum before = Momentum after

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

Students must decide which direction to take as positive and then be consistent throughout their solution. Emphasise that it is velocities not speeds (i.e. vector quantities) in the equation and so signs of terms are important.

Remind the students that coalesce means that bodies move off together as one body after the collision.

Cover lots of examples of collisions and also include examples of two particles joined by a slack light inextensible string. When the one in front moves off (or is travelling faster than the one behind), the string will get tight and 'jerk' the other particle. This 'jerk' acts as an impulse (equal and opposite on each particle). The particles will then move off together in the subsequent motion, effectively as one body, and total momentum is conserved. It is a good idea to set a variety of questions with algebraic rather than numerical values for the quantities.

During a collision each particle receives an impulse. As mentioned previously these impulses are equal in magnitude but opposite in direction. When asked for the magnitude of the impulse of A on B , make sure the students are clear that only the momentum of one particle is to be considered (and not the system as a whole). If the question refers to ' A on B ' it is probably advisable to find the impulse on B ; however, the impulse on A has the same magnitude and so it would be equally correct to find the impulse on A . Emphasise that the magnitude denotes a positive answer and so, even if the impulse turns out to be negative, the final answer must be given as positive. Note that it is also worth calculating the impulse of B on A to confirm that it is equal in magnitude but of opposite sign to the impulse of A on B .

The result will be a positive impulse if the body's velocity has been increased as a result of the collision and negative if the velocity has decreased. For example, if A were moving at -4 m s^{-1} before the collision and 3 m s^{-1} after being hit, there is a positive impulse as $3 > -4$.

A level Mathematics: Mechanics 1

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Most exam questions have an element of problem solving but an example worth considering is a gun and recoil. (Worked example below; note that units must be consistent.)

A bullet is fired from a gun with a horizontal velocity of 500 ms^{-1} . The mass of the gun is 4 kg and the mass of the bullet is 50 g . Find the initial speed of recoil of the gun.

The momentum before impact = momentum after impact

Before: Gun (4 kg) and Bullet (0.05 kg) are at rest (0 ms^{-1}).

After: Gun (4 kg) recoils at $v \text{ ms}^{-1}$ to the left, and the Bullet (0.05 kg) travels at 500 ms^{-1} to the right.

Cons. of momentum:

$$(+\leftarrow) \quad 4(0) + 0.05(0) = 4v + 0.05(-500)$$
$$\therefore 0 = 4v - 25$$
$$\therefore 25 = 4v$$
$$\therefore v = 6.25 \text{ ms}^{-1}$$

To extend the work on collisions you could include some questions involving successive collisions between pairs of spheres (e.g. A hits B then B hits C). Note that when a sphere hits a wall it is possible to find the impulse on the sphere (using change in momentum) but remember to take account of the change in direction of motion due to the impact.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Questions on momentum in one-dimension are generally completed well. The main reason for loss of marks is arithmetic errors or sign errors. It should be remembered that both speed and magnitude of impulse are positive (scalar) quantities.

Occasionally students forget to take account of the directions, particularly when considering impulses, and so run into difficulties. Remind students that any direction of motion should be described in the context of the problem (such as 'direction reversed' or 'opposite direction') and not relative to their diagram (such as 'to the right' or by drawing an arrow).

A level Mathematics: Mechanics 1

4d. Friction forces (including coefficient of friction μ) (4.4)

Teaching time

4 hours

OBJECTIVES

By the end of the sub-unit, students should:

- understand that a rough plane will have an associated frictional force, which opposes relative motion (i.e. the direction of the frictional force is always opposite to how the object is moving or 'wants' to move);
- understand that the 'roughness' of two surfaces is represented by a value called the coefficient of friction represented by μ ;
- know that $0 \leq \mu$ but that there is no theoretical upper limit for μ although for most surfaces it tends to be less than 1 and that a 'smooth' surface has a value of $\mu = 0$;
- be able to draw force diagrams and solve problems involving rough surfaces which include the frictional force.

TEACHING POINTS

Start by asking students to rub their hands together vigorously. The warmth is caused by microscopic peaks and troughs on the surface of the skin interlocking. The rougher the surface, the 'sharper' these peaks and troughs. Explain to students that this principle applies even to the smoothest looking surfaces and the force which opposes motion is called the frictional force. The value which represents the roughness is called the coefficient of friction (μ) and is zero for a smooth surface.

If we consider a book on a rough horizontal table, it will be *harder* to move the book if:

- we put a 'paperweight' on it (increasing the reaction force)

or

- we put it on a rougher surface (increasing the value of μ).

Therefore the expression to model frictional forces uses these two factors (in direct proportion) and is given by μR . The direction of the frictional force acts in the direction opposite to the direction of movement of the object. The coefficient of friction (μ) for any rough surface is always positive and although there is no theoretical upper limit it tends to be less than 1 for most surfaces.

Present some simple problems involving moving particles and frictional forces and get students to draw force diagrams. Cover problems involving particles moving along rough horizontal surfaces and up or down rough inclined planes.

OPPORTUNITIES FOR PROBLEM SOLVING/MODELLING

A variety of more complex problems can be given to students to consolidate the various scenarios presented in this unit. These could include a pulley system along a rough inclined plane with one particle on the inclined plane and the other hanging freely. Students should be given plenty of practice using $F = ma$ and resolving forces.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Students are often good at drawing force diagrams, but common errors are omitting arrowheads, incorrectly labelling (e.g. 4 kg rather than 4g) and missing off the normal reaction or friction forces. Students can sometimes struggle to work out the direction of the frictional force.

Some students may mistakenly think that the coefficient of friction changes if the mass of an object or the angle of the slope changes.

A level Mathematics: Mechanics 1

NOTES

Many problems in mechanics involve resolving forces, so students should be given plenty of practice at doing this. It is also a skill that is required in the next unit, Statics.

A level Mathematics: Mechanics 1

UNIT 5: Statics of a particle

Equilibrium, forces in vector form, maximum value of the frictional force (5.1)(5.2)(5.3)

Teaching time

4 hours

[Return to overview](#)

SPECIFICATION REFERENCES

5.1 Forces treated as vectors. Resolution of force

5.2 Equilibrium of a particle under coplanar forces. Weight, normal reaction, tension and thrust, friction

5.3 Coefficient of friction

PRIOR KNOWLEDGE

- Basic vectors, magnitude and direction (see Unit 2)
- **i**, **j** vectors (see Unit 2)
- Force diagrams and assumptions (see Units 1 and 4)
- Resolution of forces (see Unit 4)

IAS Mathematics – P1 content

- 2D trigonometry
- Cosine and sine rules
- $\frac{\sin x}{\cos x} = \tan x$ (to find the angle of the resultant)

KEYWORDS

Force, weight, tension, thrust, reaction, resultant, magnitude, direction, force diagram, equilibrium, inextensible, light, negligible, particle, smooth, rough, perpendicular.

OBJECTIVES

By the end of the sub-unit, students should:

- be able to find unknown forces acting on a particle which remains in equilibrium;
- understand how to find the resultant force (magnitude and direction);
- be able to find the resultant of several concurrent forces by vector addition;
- be able to resolve a force into components and be able to select suitable directions for resolution;
- be able to solve problems involving friction by using the relationship $F \leq \mu R$.

TEACHING POINTS

Begin by reviewing force diagrams and resolving forces in two perpendicular directions. Point out that if an object is in equilibrium then the sum of the components of the forces in each direction is equal to zero. Consider problems where an unknown force in a given direction has to be found. Review the **i–j** notation and look at examples where forces are given in **i–j** form or as column vectors. Point out that the vector sum of the forces is equal to zero when the object is in equilibrium. Link this to the vector addition rule. To find the resultant, R , of two or more forces start by considering two forces acting at right angles to one another (horizontal and vertical), use Pythagoras and trigonometry to find the hypotenuse (resultant R) and angle (direction θ above the horizontal) respectively. You could link the resultant to the idea of velocity from speed.

A level Mathematics: Mechanics 1

Look at two forces acting at *any* angle and show that the triangle can be solved using the cosine rule (to find the resultant) and sine rule (to find the direction). Look at problems such as finding the magnitude and direction of the force exerted on a pulley.

Move on to problems involving friction. Consider what happens if someone wants to pull a very heavy weight along a rough surface. Discuss the idea that as the force exerted increases the friction must also increase until a maximum resistance is reached. The inequality $F \leq \mu R$ applies until the force wanting to cause motion reaches the limiting value μR , called limiting friction.

Consider a 10 kg book on a rough horizontal plane. If $\mu = 0.5$, investigate the value of the frictional force if the pushing force, P is **a** 10 N, **b** 98 N, **c** 100 N.

Now place the book on an inclined plane and analyse the limiting friction being careful to stress that the reaction force is NOT the weight in this case. Will the book begin to slide for different angles of plane? What is the maximum angle achievable before the book slides?

OPPORTUNITIES FOR PROBLEM SOLVING/MODELLING

This topic can be linked with Unit 4 and these techniques used to solve statics and dynamics problems.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

When resolving common errors are: to omit g ; sign errors; reversal or confusion* of when to use cos and/or sin; to omit one force (usually weight).

Students may also easily get confused by the vocabulary and mix up 'resultant' and 'reaction'.

NOTES

For students who find these concepts difficult it is possible to simplify most questions by restricting the resolving of a force to using *just* $\cos \theta$. This can be done by using the method of '**cos across** the number of degrees the force has to be turned to reach the direction we want to resolve in'.

*This eliminates the choice of cos or sin for weaker students and can avoid the confusion mentioned in the paragraph above.

A level Mathematics: Mechanics 1

UNIT 6: Moments Forces' turning effects (9.1)

Teaching time
7 hours

[Return to overview](#)

SPECIFICATION REFERENCES

6.1 Moment of a force

PRIOR KNOWLEDGE

- Types of forces and force diagrams (see Units 4 and 5)
- Assumptions made throughout this course; S.I. units (see throughout, but especially Unit 1)
- Weight = mass $\times$ gravity ($W = mg$) (see Unit 4)
- Basic equilibrium (see Unit 5)

IGCSE/GCSE (9-1) in Mathematics at Higher Tier

- Solving linear and simultaneous equations

KEYWORDS

Moment, turning effect, sense, newton metre (N m), equilibrium, reaction, tension, rod, uniform, non-uniform, centre of mass, resolve, tilting, 'on the point', concurrent.

OBJECTIVES

By the end of this unit, students should:

- realise that a force can produce a turning effect;
- know that a moment of a force is given by the formula force $\times$ distance giving N m and know what the sense of a moment is;
- understand that the force and distance must be perpendicular to one another;
- be able to draw mathematical models to represent horizontal rod problems;
- realise what conditions are needed for a system to remain in equilibrium;
- be able to solve problems when a bar is on the point of tilting.

TEACHING POINTS

Start by asking two students to push up and down equally on two points of a ruler (or rod/beam) which are directly above or below each other. The forces balance and if we resolve vertically, the resultant force is zero. Hence the ruler will not move (equilibrium). However, if the two positions are separated, the ruler will turn, despite the forces still having no resultant in the vertical direction. So if two (or more) forces are not concurrent, there may be a turning effect. (See diagrams below.)



Next think about a door handle and imagine it was moved nearer the hinge of the door. Common sense tells us the door will be harder to open or close, so any formula for the turning effect of forces must involve distance as well as force.

A level Mathematics: Mechanics 1

Show a bicycle pedal in different positions and discuss which one makes turning easier. (See diagrams below.)



A discussion around this can lead to the understanding that the moment of a force, is a measure of its turning effect and is given by the formula:

$$\text{moment of a force about a point} = \text{force } (F) \times \text{perpendicular distance from the point to the line of action of the force } (d)$$

The unit is newton metres, N m.

Ask students questions such as: How do we work out the distance, d , in the second bicycle pedal diagram? What additional information do we need? What if the pedal was at the topmost point, vertically above the axle?

The force and distance must be perpendicular to one another, but in this unit we will only be considering horizontal bars, supported or suspended by reactions and tensions respectively. These forces will naturally be vertical and parallel to one another, so the moments formula can be applied easily and the only thing to consider is the *sense* of the moment (whether the turning effect of each force is clockwise (negative) or anticlockwise (positive)).

Demonstrate that a uniform ruler will balance about its centre (where all the weight acts) and that this central point is therefore its centre of mass. Use this to extend students' basic idea of equilibrium as a system where there is no resultant force and also no overall turning effect, i.e. $R(\uparrow) = 0$ N and the sum of the moments = 0 N m.

Make sure all the assumptions are revisited from earlier in the course e.g. model a rod as a straight line, a person standing on a bridge as a particle, strings being inextensible etc. The centre of mass is at the centre of the rod only if it is stated as being uniform.

Before starting on questions, make sure students know the notation: when we 'take moments' about a certain point (say A), we write this as $M(A)$.

Cover questions that involve:

- rods resting on two or more supports
- a rod which is suspended at two or more points
- finding the position of the centre of mass of a non-uniform rod.

Make sure you stress that theoretically we can take moments about any point and, together with resolving (vertically), we can solve any problem. However, some positions will make the solution more efficient and subsequently involve less algebra. Show students that taking moments about a point through which a force acts is zero as the distance to that force is zero.

OPPORTUNITIES FOR PROBLEM SOLVING/MODELLING

Tilting problems: consider rods on the point of tilting. (You could demonstrate with a ruler resting on a couple of erasers, then add some coins to the end until it lifts off one of the supports.) The forces still remain vertical and the rod horizontal (just) as the rod tends to want to lift. One of the reactions 'becomes' zero at the point of tilting.

A level Mathematics: Mechanics 1

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Many students made their life more difficult than necessary by not taking the easy resolving option and using two moments equations resulting in simultaneous equations which can be difficult to solve.

Clear diagrams can help to overcome some errors such as using distances from the wrong point or missing forces (often the weight).

Students should also be reminded to read the question carefully and give their answer in the correct form – being particularly careful not to mix up weight and mass.

NOTES

This topic forms a good introduction to resolving and equilibrium as the forces are confined to only vertical examples.